

## Binomial Probabilities - Part II



## Preliminaries and Objectives

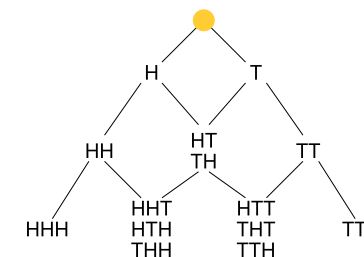
### Preliminaries

- Basic Probability (AND, OR, NOT)
- Pascal's Triangle
- Combinations
- Binomial Probabilities  $p = 1/2$
- Random Variables

### Objectives

- Calculate probabilities in successive trials with only two outcomes, either succeed or fail.

## Flip three coins



$$P(H = 3) = \frac{1}{8}, \quad P(H = 2) = \frac{3}{8}, \quad P(H = 1) = \frac{3}{8}, \quad P(H = 0) = \frac{1}{8}$$

## Unequally likely outcomes

A basketball player shoots 70% from the free throw line. Let  $X$  = the number of shots made on two attempts.

$X$ =	outcomes	probability
0	oo	$(.3)(.3) = .09$
1	o•, •o	$(.7)(.3) + (.3)(.7) = .42$
2	••	$(.7)(.7) = .49$

## Unequally likely outcomes

A basketball player shoots 70% from the free throw line. Let  $X$  = the number of shots made on five attempts. What is the probability that three of the five shots are made?

••••o   •••o•   ••o••   •o•••   •o•••  
 •o•••   o••••   o••••   o••••   o••••

$$P(\bullet\bullet\bullet\bullet o) = P(\bullet\bullet\bullet o\bullet) = \dots$$

$$(.7)(.7)(.7)(.3)(.3) = (.7)(.7)(.3)(.7)(.3) = (.7)^3(.3)^2$$

$$P(X = 3) = \binom{5}{3} (.7)^3 (.3)^2 = .3087$$

## Binomial Probability Formula

Let  $p$  = probability of success on a single trial.

Let  $q = 1 - p$  = probability of failure on a single trial.

Let  $X$  be a random variable that counts the number of successes in  $n$  independent trials.

### Binomial Probability

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

where  $\binom{n}{k}$  is the  $k^{\text{th}}$  entry in row  $n$  of Pascal's Triangle.

## Exercise

Find the probability that a 65% free throw shooter will make 3 of 4 attempts.

### Solution:

$$P(X = 3) = \binom{4}{3} (.65)^3 (.35) = 4(.65)^3 (.35) = .384475$$