The Binomial Distribution and the Bell Curve

Preliminaries and Objectives
- Pascal's Triangle
- Binomial Theorem
- Probability
- Binomial Distributions
- Random Variables

Objectives
- Determine the percentage of a (normally distributed) population within a given range
- Confidence Intervals

The Bell Curve
- \( \mu \) = average value (mean)
- \( \sigma \) = standard deviation

The Bell Curve - a.k.a. Normal Distribution

\[ P(X < \mu) = \frac{1}{2} \]

\( \mu \) = average value (mean)

\[ P(\mu - \sigma < X < \mu + \sigma) \approx 0.6827 \]

\[ P(X < \mu - \sigma) \approx 0.1587 \]
\[ P(X > \mu - \sigma) \approx 0.8413 \]
z-scores

The z-score or z-value is a measure of the number of standard deviations above or below average.

\[ z = \frac{X - \mu}{\sigma} \]

Example: In a population, the mean \( \mu = 162 \) with standard deviation \( \sigma = 7 \), what is the z-score for a measurement of \( X = 148 \)?

\[ z = \frac{148 - 162}{7} = -2 \]

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The mean female height is 162 cm with a standard deviation 7 cm. What is the probability that a randomly chosen female is shorter than 148 cm?

\[ z = \frac{148 - 162}{7} = -2 \]

\[ P(X < 148) = P(z < -2) = 0.02275 \]

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If \( X \) has a normal distribution with \( \mu = 11 \) and \( \sigma = 5 \), what is \( P(X > 14) \)?

\[ z = \frac{14 - 11}{5} = 0.6 \]

\[ P(X > 14) = P(z < 0.6) = 0.72575 \]

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In a normally distributed population, with \( \mu = 100 \) and \( \sigma = 10 \), find an interval \( a < X < b \), such that 95% of the population falls in the interval.

\[ P(z < -1.96) = 0.025 \text{ and } P(z < 1.96) = 0.975 \text{ so we need} \]

\[ -1.96 < z < 1.96 \]

\[ z = -1.96 \Rightarrow \frac{x_{\text{min}} - \mu}{\sigma} = -1.96 \Rightarrow \frac{x_{\text{min}} - 100}{10} = -1.96 \]

\[ \Rightarrow x_{\text{min}} = 80.4 \]

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90% confidence interval \(-1.645 < z < 1.645\)

95% confidence interval \(-1.960 < z < 1.960\)

99% confidence interval \(-2.576 < z < 2.576\)

We wish to design a bicycle that adjusts so that 99% of the population can ride comfortably. \( \mu = 162 \), \( \sigma = 7 \). Find an interval that contains 99% of the population.

99% confidence interval \( \Rightarrow -2.576 < z < 2.576 \)

\[ z = \pm 2.576 \Rightarrow \frac{X - 162}{7} = \pm 2.576 \Rightarrow X = 162 \pm 2.576(7) \Rightarrow X = 162 \pm 18.03 \]

99% of the population lies in the interval \( 143.97 < X < 180.03 \)
Example 4

30% of parts in a manufacturing process are defective, with the other 70% being useable. A shipment of 1000 parts will, on average have $\mu = 700$ useable parts, with a standard deviation of $\sigma = 14.5$. Find a 90% confidence interval for the number of useable parts.

90% confidence interval $\Rightarrow -1.645 < z < 1.645$

**Solution:**

$$z = \pm 1.645 \Rightarrow \frac{X - 700}{14.5} = \pm 1.645$$

$\Rightarrow X = 700 \pm 1.645(14.5) \Rightarrow X = 700 \pm 23.8$

90% of the population lies in the interval $676 < X < 724$