

Expected Value



Preliminaries and Objectives

Preliminaries

- Average
- Probability of events
- Sigma Notation
- Binomial Theorem

Objectives

- Calculate the Average Value (Expected Value) of a random variable

Calculating an Average

If during a 30-day month, you worked for 8 hours on 13 of those days, 6 hours on 3 of those days, 4 hours on 4 of those days and had 10 days off, how many hours per day did you work on average?

$$\frac{0(10) + 8(13) + 6(3) + 4(4)}{30} = 4.60$$

$$0 \cdot \left(\frac{10}{30}\right) + 8 \cdot \left(\frac{13}{30}\right) + 6 \cdot \left(\frac{3}{30}\right) + 4 \cdot \left(\frac{4}{30}\right) = 4.60$$

Definition













If a random variable X , takes on possible values v_1, v_2, v_3, \dots which have probabilities p_1, p_2, p_3, \dots respectively, then the **expected value*** of X is

$$E(X) = v_1 \cdot p_1 + v_2 \cdot p_2 + v_3 \cdot p_3 + \dots$$

$$E(X) = \sum_i v_i \cdot p_i$$

* Note: **Expected value** is also called **average value**. In statistics, it is referred to as the **mean**.

Example 1 - Two dice

						
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

Example 1 - Two dice

k	$P(X = k)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

Example 1 - Two dice

$$\begin{aligned}
 E(X) &= (2)\frac{1}{36} + (3)\frac{2}{36} + (4)\frac{3}{36} + (5)\frac{4}{36} + (6)\frac{5}{36} + (7)\frac{6}{36} \\
 &\quad + (8)\frac{5}{36} + (9)\frac{4}{36} + (10)\frac{3}{36} + (11)\frac{2}{36} + (12)\frac{1}{36} \\
 &= 7
 \end{aligned}$$

Example 2 - Flipping 4 coins

k	$P(X = k)$
0	$\frac{1}{16}$
1	$\frac{4}{16}$
2	$\frac{6}{16}$
3	$\frac{4}{16}$
4	$\frac{1}{16}$

$$E(X) = (0)\frac{1}{16} + (1)\frac{4}{16} + (2)\frac{6}{16} + (3)\frac{4}{16} + (4)\frac{1}{16} = 2$$

Example 3 - Is this game fair?

k	$P(X = k)$
-\$1	.70
+\$1	.20
+\$4	.10

$$E(X) = (-1)(.70) + (1)(.20) + (4)(.10) = -.10$$