

Binomial Probabilities - Part II



Preliminaries and Objectives

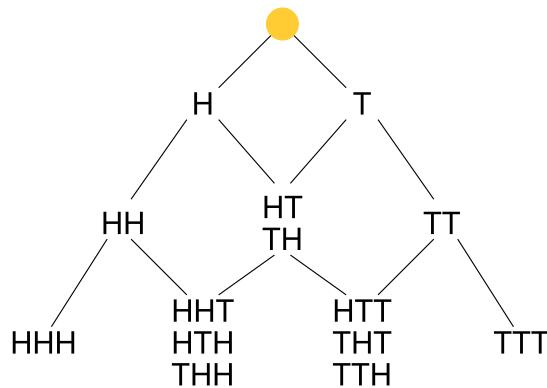
Preliminaries

- Basic Probability (AND, OR, NOT)
- Pascal's Triangle
- Combinations
- Binomial Probabilities $p = 1/2$
- Random Variables

Objectives

- Calculate probabilities in successive trials with only two outcomes, either succeed or fail.

Flip three coins



$$P(H = 3) = \frac{1}{8}, \quad P(H = 2) = \frac{3}{8}, \quad P(H = 1) = \frac{3}{8}, \quad P(H = 0) = \frac{1}{8}$$

Unequally likely outcomes

A basketball player shoots 70% from the free throw line. Let X = the number of shots made on two attempts.

$X =$	outcomes	probability
0	oo	$(.3)(.3) = .09$
1	•o, o•	$(.7)(.3) + (.3)(.7) = .42$
2	••	$(.7)(.7) = .49$

Unequally likely outcomes

A basketball player shoots 70% from the free throw line. Let X = the number of shots made on five attempts. What is the probability that three of the five shots are made?

●●●○○ ●●○○● ●●○○● ●○○●○ ●○○●●
●○○●● ○●●●○ ○●●●● ○●●●● ○○○●●

$$P(\bullet\bullet\bullet\circ\circ) = P(\bullet\bullet\circ\bullet\circ) = \dots \\ (.7)(.7)(.7)(.3)(.3) = (.7)(.7)(.3)(.7)(.3) = (.7)^3(.3)^2$$

$$P(X = 3) = \binom{5}{3}(.7)^3(.3)^2 = .3087$$

Binomial Probability Formula

Let p = probability of success on a single trial.

Let $q = 1 - p$ = probability of failure on a single trial.

Let X be a random variable that counts the number of successes in n independent trials.

Binomial Probability

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

where $\binom{n}{k}$ is the k^{th} entry in row n of Pascal's Triangle.

Exercise

Find the probability that a 65% free throw shooter will make 3 of 4 attempts.

Solution:

$$P(X = 3) = \binom{4}{3} (.65)^3 (.35) = 4(.65)^3 (.35) = .384475$$