Preliminaries and Objectives

Preliminaries
- Pascal's Triangle
- Binomial Theorem
- Probability
- Binomial Distributions
- Random Variables

Objectives
- Determine the percentage of a (normally distributed) population within a given range
- Confidence Intervals

The Binomial Distribution and the Bell Curve

The Bell Curve

\[
\mu = \text{average value (mean)} \\
\sigma = \text{standard deviation}
\]

Binomial Distributions

Flip 4 coins

TTTT TTTH TTHH THHH HHHH
Binomial Distributions

The Bell Curve - a.k.a. Normal Distribution

Binomial Distributions

The Bell Curve - a.k.a. Normal Distribution

Flip 10 coins

Flip 1000 coins

$P(X < \mu) = \frac{1}{2}$

$\mu = \text{average value (mean)}$

$P(\mu - \sigma < X < \mu + \sigma) \approx 0.6827$
**The Bell Curve - a.k.a. Normal Distribution**

![Bell Curve Diagram]

**z-scores**

The z-score or z-value is a measure of the number of standard deviations above or below average.

\[ z = \frac{X - \mu}{\sigma} \]

Example: In a population, the mean \( \mu = 162 \) with standard deviation \( \sigma = 7 \), what is the z-score for a measurement of \( X = 148 \)?

\[ z = \frac{148 - 162}{7} = -2 \]

**Normal distribution table**

<table>
<thead>
<tr>
<th>z-score</th>
<th>( P(X &lt; z) )</th>
<th>z-score</th>
<th>( P(X &lt; z) )</th>
<th>z-score</th>
<th>( P(X &lt; z) )</th>
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<tbody>
<tr>
<td>0.0</td>
<td>0.50000</td>
<td>1.0</td>
<td>0.84134</td>
<td>2.0</td>
<td>0.97725</td>
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<td>1.1</td>
<td>0.86433</td>
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<td>0.98214</td>
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<tr>
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<td>2.2</td>
<td>0.98610</td>
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<td>0.90320</td>
<td>2.3</td>
<td>0.98928</td>
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<tr>
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<td>0.99180</td>
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<td>0.93319</td>
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<td>0.99379</td>
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<td>0.99534</td>
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<td>0.95543</td>
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<td>2.9</td>
<td>0.99813</td>
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<tr>
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<td>2.0</td>
<td>0.97725</td>
<td>3.0</td>
<td>0.99865</td>
</tr>
</tbody>
</table>

**Example 1**

The mean female height is 162 cm with a standard deviation 7 cm. What is the probability that a randomly chosen female is shorter than 148 cm?

\[ z = \frac{148 - 162}{7} = -2 \]

\[ P(X < 148) = P(z < -2) \approx 0.02275 \]
Example 2

If $X$ has a normal distribution with $\mu = 11$ and $\sigma = 5$, what is $P(X > 14)$?

$$z = \frac{14 - 11}{5} = 0.6$$

$$P(X < 14) = P(z < 0.6) \approx 0.72575$$

$$P(X > 14) = 1 - P(X < 14) \approx 0.27425$$

Confidence Intervals

In a normally distributed population, with $\mu = 100$ and $\sigma = 10$, find an interval $a < X < b$, such that 95% of the population falls in the interval.

$$P(z < -1.96) \approx 0.025 \text{ and } P(z < 1.96) \approx 0.975 \text{ so we need } -1.96 < z < 1.96$$

$$z = -1.96 \Rightarrow \frac{X_{\text{min}} - \mu}{\sigma} = -1.96 \Rightarrow \frac{X_{\text{min}} - 100}{10} = -1.96$$

$$\Rightarrow X_{\text{min}} - 100 = -19.6 \Rightarrow X_{\text{min}} = 80.4$$

Confidence Intervals

In a normally distributed population, with $\mu = 100$ and $\sigma = 10$, find an interval $a < X < b$, such that 95% of the population falls in the interval.

$$P(z < -1.96) \approx 0.025 \text{ and } P(z < 1.96) \approx 0.975 \text{ so we need } -1.96 < z < 1.96$$

$$z = 1.96 \Rightarrow \frac{X_{\text{max}} - \mu}{\sigma} = 1.96 \Rightarrow \frac{X_{\text{max}} - 100}{10} = 1.96$$

$$\Rightarrow X_{\text{max}} - 100 = 19.6 \Rightarrow X_{\text{max}} = 119.6$$

$$P(80.4 < X < 119.6) = 95%$$
**Example 3**

We wish to design a bicycle that adjusts so that 99% of the population can ride comfortably. $\mu = 162$, $\sigma = 7$. Find an interval that contains 99% of the population.

99% confidence interval $\Rightarrow -2.576 < z < 2.576$

$$z = \pm 2.576 \Rightarrow \frac{X - 162}{7} = \pm 2.576 \Rightarrow X = 162 \pm 2.576(7) \Rightarrow X = 162 \pm 18.03$$

99% of the population lies in the interval $143.97 < X < 180.03$

**Example 4**

30% of parts in a manufacturing process are defective, with the other 70% being usable. A shipment of 1000 parts will, on average have $\mu = 700$ useable parts, with a standard deviation of $\sigma = 14.5$. Find a 90% confidence interval for the number of useable parts.

90% confidence interval $\Rightarrow -1.645 < z < 1.645$

**Solution:**

$$z = \pm 1.645 \Rightarrow \frac{X - 700}{14.5} = \pm 1.645$$

$$X = 700 \pm 1.645(14.5) \Rightarrow X = 700 \pm 23.8$$

90% of the population lies in the interval $676 < X < 724$