The Binomial Distribution and the Bell Curve

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Preliminaries and Objectives

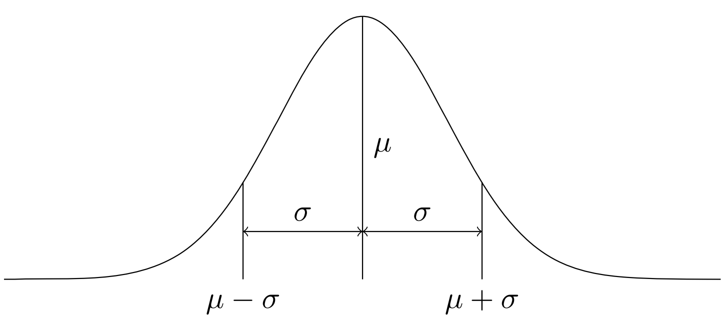
Preliminaries

- Pascal's Triangle
- Binomial Theorem
- Probability
- Binomial Distributitions
- Random Variables

Objectives

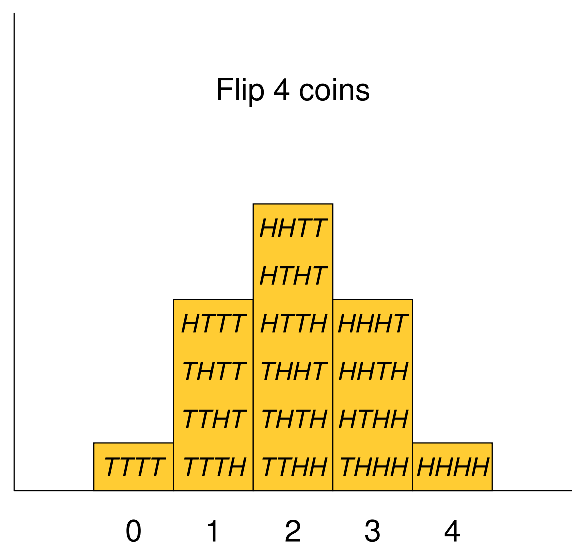
- Determine the percentage of a (normally distributed) population within a given range
- Confidence Intervals

The Bell Curve

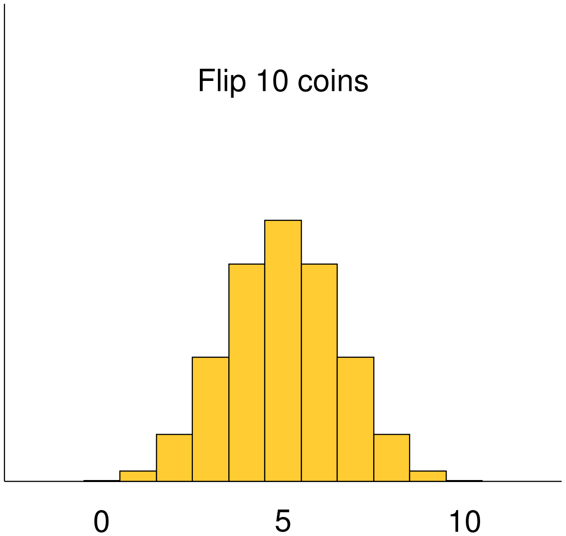


$\mu =$ average value (mean)
$\sigma =$ standard deviation

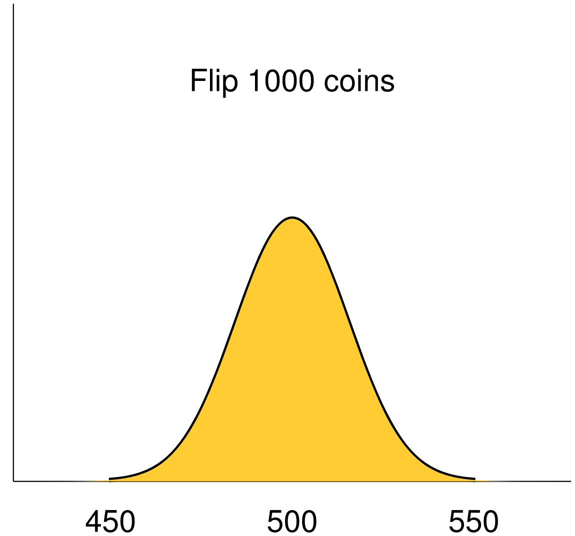
Binomial Distributions



Binomial Distributions

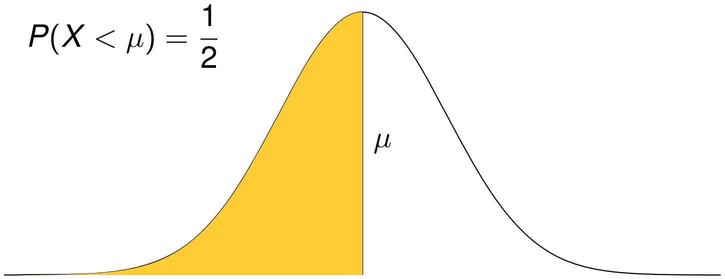


Binomial Distributions



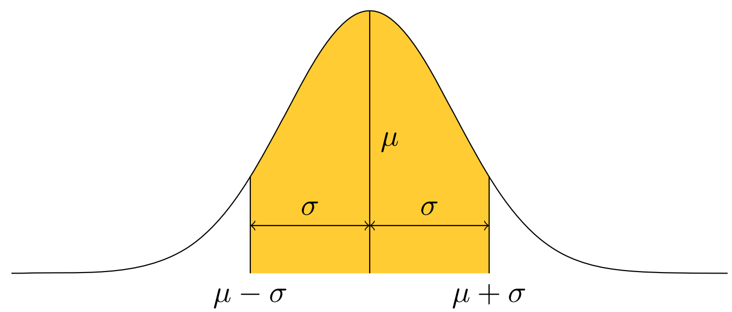
The Bell Curve - a.k.a. Normal Distribution

$$P(X < \mu) = \frac{1}{2}$$



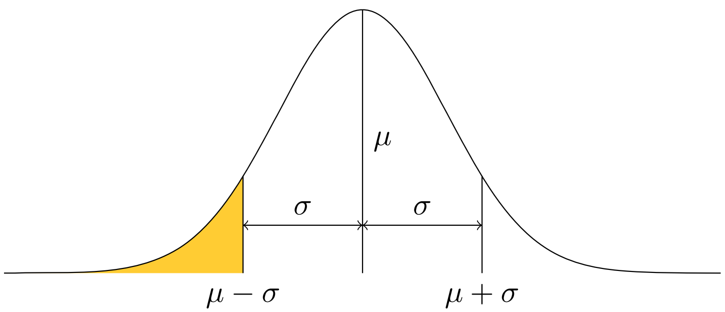
$\mu =$ average value (mean)

The Bell Curve - a.k.a. Normal Distribution



$$P(\mu - \sigma < X < \mu + \sigma) \approx 0.6827$$

The Bell Curve - a.k.a. Normal Distribution



$$P(X < \mu - \sigma) \approx 0.1587$$

$$P(X > \mu - \sigma) \approx 0.8413$$

z-scores

The *z*-score or *z*-value is a measure of the number of standard deviations above or below average.

$$z = \frac{X - \mu}{\sigma}$$

Example: In a population, the mean $\mu = 162$ with standard deviation $\sigma = 7$, what is the *z*-score for a measurement of $X = 148$?

$$z = \frac{148 - 162}{7} = -2$$

Normal distribution table

z-score	$P(X < z)$	*z*-score	$P(X < z)$	*z*-score	$P(X < z)$
0.0	0.50000	1.0	0.84134	2.0	0.97725
0.1	0.53983	1.1	0.86433	2.1	0.98214
0.2	0.57926	1.2	0.88493	2.2	0.98610
0.3	0.61791	1.3	0.90320	2.3	0.98928
0.4	0.65542	1.4	0.91924	2.4	0.99180
0.5	0.69146	1.5	0.93319	2.5	0.99379
0.6	0.72575	1.6	0.94520	2.6	0.99534
0.7	0.75804	1.7	0.95543	2.7	0.99653
0.8	0.78814	1.8	0.96407	2.8	0.99744
0.9	0.81594	1.9	0.97128	2.9	0.99813
1.0	0.84134	2.0	0.97725	3.0	0.99865

Example 1

The mean female height is 162 cm with a standard deviation 7 cm. What is the probability that a randomly chosen female is shorter than 148 cm?

$$z = \frac{148 - 162}{7} = -2$$

$$P(X < 148) = P(z < -2) \approx 0.02275$$

Example 2

If X has a normal distribution with $\mu = 11$ and $\sigma = 5$, what is $P(X > 14)$?

$$z = \frac{14 - 11}{5} = 0.6$$

$$P(X < 14) = P(z < 0.6) \approx 0.72575$$

$$P(X > 14) = 1 - P(X < 14) \approx 0.27425$$

Confidence Intervals

In a normally distributed population, with $\mu = 100$ and $\sigma = 10$, find an interval $a < X < b$, such that 95% of the population falls in the interval.

$P(z < -1.96) \approx 0.025$ and $P(z < 1.96) \approx 0.975$ so we need $-1.96 < z < 1.96$

$$z = -1.96 \Rightarrow \frac{X_{min} - \mu}{\sigma} = -1.96 \Rightarrow \frac{X_{min} - 100}{10} = -1.96$$
$$\Rightarrow X_{min} - 100 = -19.6 \Rightarrow x_{min} = 80.4$$

Confidence Intervals

In a normally distributed population, with $\mu = 100$ and $\sigma = 10$, find an interval $a < X < b$, such that 95% of the population falls in the interval.

$P(z < -1.96) \approx 0.025$ and $P(z < 1.96) \approx 0.975$ so we need $-1.96 < z < 1.96$

$$z = 1.96 \Rightarrow \frac{X_{max} - \mu}{\sigma} = 1.96 \Rightarrow \frac{X_{max} - 100}{10} = 1.96$$
$$\Rightarrow X_{max} - 100 = 19.6 \Rightarrow x_{max} = 119.6$$

Confidence Intervals

In a normally distributed population, with $\mu = 100$ and $\sigma = 10$, find an interval $a < X < b$, such that 95% of the population falls in the interval.

$P(z < -1.96) \approx 0.025$ and $P(z < 1.96) \approx 0.975$ so we need $-1.96 < z < 1.96$

$$P(80.4 < X < 119.6) = 95\%$$

Standard Confidence Intervals

90% confidence interval $-1.645 < z < 1.645$

95% confidence interval $-1.960 < z < 1.960$

99% confidence interval $-2.576 < z < 2.576$

Example 3

We wish to design a bicycle that adjusts so that 99% of the population can ride comfortably. $\mu = 162$, $\sigma = 7$. Find an interval that contains 99% of the population.

99% confidence interval $\Rightarrow -2.576 < z < 2.576$

$z = \pm 2.576 \Rightarrow \dfrac{X - 162}{7} = \pm 2.576 \Rightarrow X = 162 \pm 2.576(7) \Rightarrow$
$X = 162 \pm 18.03$

99% of the population lies in the interval $143.97 < X < 180.03$

Example 4

30% of parts in a manufacturing process are defective, with the other 70% being useable. A shipment of 1000 parts will, on average have $\mu = 700$ useable parts, with a standard deviation of $\sigma = 14.5$. Find a 90% confidence interval for the number of useable parts.

90% confidence interval $\Rightarrow -1.645 < z < 1.645$

Solution:
$z = \pm 1.645 \Rightarrow \dfrac{X - 700}{14.5} = \pm 1.645$

$\Rightarrow X = 700 \pm 1.645(14.5) \Rightarrow X = 700 \pm 23.8$

90% of the population lies in the interval $676 < X < 724$