

Bayes Rule and Testing



Preliminaries and Objectives

Preliminaries

- Probability of Events

Objectives

- Find the probability that a test subject is positive given that they tested positive.

Dragon Pox epidemic

In a community of 10,000 people, 2% of the population is infected with dragon pox. There is a test to determine whether or not an individual has dragon pox that is quite accurate. If you have dragon pox, the test will correctly determine that 99% of the time. It will read a 'false negative' only 1% of the time. Similarly, if you do not have the disease, the test will correctly determine that 97% of the time and will give a 'false positive' reading to 3% of disease-free individuals. If you test positive for dragon pox, how likely are you to actually have the disease?

Bayes Rule

| <i>pop.</i> = 10000 | Tested positive | Tested negative | |
|---------------------|-----------------|-----------------|----------|
| infected | 99% 198 | 1% 2 | 2% 200 |
| not infected | 3% 294 | 97% 9506 | 98% 9800 |
| Total | 492 | 9508 | 10000 |

$$P(\text{infected} \mid +) = \frac{198}{492} \approx 40.2\%$$

Bayes Rule

| | Tested positive | Tested negative | |
|--------------|---------------------------|---------------------------|------|
| infected | $(.99)(.02)$ $= .0198$ | $(.01)(.02)$ $= .0002$ | 0.02 |
| not infected | $(.03)(.98)$ $= .0294$ | $(.97)(.98)$ $= .9506$ | 0.98 |
| Total | .0492 | .9508 | |

$$P(\text{infected} | +) = \frac{0.0198}{0.0492} \approx 40.2\%$$