Unions, Intersections, and Complements in Probability

Preliminaries and Objectives

Preliminaries

- Techniques of Counting
- Definition of Probability

Objectives

- Find probabilities of events combined using AND, OR, NOT

Unions

Example: Roll two dice. What is the probability that the total of the two dice is either 7 or 11?

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Unions

Example: Roll two dice. What is the probability that the total of the two dice is either 7 or 11?

E = total is 7 $\quad P(E) = \frac{6}{36} \quad$ E and F are mutually exclusive

F = total is 11 $\quad P(F) = \frac{2}{36}$

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36}$$

Conditional Probability

Sometimes the occurrence of an event changes our mind about the probability of another event.

E = roll a ⚅

F = roll ≥ 10 on two dice

$P(E) = \frac{1}{6}$

$P(F) = \frac{6}{36}$

⚅⚃ ⚄⚄ ⚃⚅ ⚅⚄ ⚄⚅ ⚅⚅

Conditional Probability

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$P(E) = \frac{1}{6}$

$P(F) = \frac{6}{36}$

⚅⚃ ⚅⚄ ⚅⚅

$P(F \mid E) = \frac{3}{6}$

Conditional Probability

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Independence

	⚀	⚁	⚂	⚃	⚄	⚅
⚀						
⚁						
⚂						
⚃						
⚄						
⚅						

If $P(F \mid E) = P(F)$, then E and F are independent.

Intersections

If the occurrence of event E has no effect on the occurrence of event F, then E and F are said to be **independent**.

When rolling two dice, what is the probability that both are ⚄ ?

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Intersections

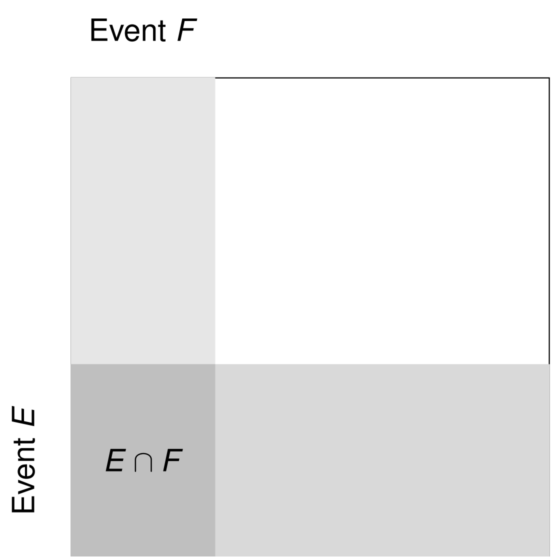
When rolling two dice, what is the probability that both are ⚄ ?

$E =$ ⚄
$F =$ ⚄

$P(E \text{ and } F) = P(E \cap F) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$P(E \text{ and } F) = P(E \cap F) = P(E) \cdot P(F)$

Intersections



Complements

There is a 40% chance that it will rain today. What is the chance that it will not rain today?

Answer: $P(\text{no rain}) = 1 - P(\text{rain}) = 60\%$

Unions of Independent Events

When rolling two dice, what is the probability that at least one of the dice is ⚄?

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$$

Examples

When picking a card from a standard deck, what is the probability that ...

1. the card is either a ♠ or ♣?
2. the card is 7 and ♡?
3. the card is not a King?
4. the card is either a 7 or a ♠?

K♠	K♥	K♦	K♣
Q♠	Q♥	Q♦	Q♣
J♠	J♥	J♦	J♣
10♠	10♥	10♦	10♣
9♠	9♥	9♦	9♣
8♠	8♥	8♦	8♣
7♠	7♥	7♦	7♣
6♠	6♥	6♦	6♣
5♠	5♥	5♦	5♣
4♠	4♥	4♦	4♣
3♠	3♥	3♦	3♣
2♠	2♥	2♦	2♣
A♠	A♥	A♦	A♣

Examples

When picking a card from a standard deck, what is the probability that ...

1. the card is either a ♠ or ♣?
2. the card is 7 and ♡?
3. the card is not a King?
4. the card is either a 7 or a ♠?

Answers:

1. $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$
2. $\frac{1}{13} \cdot \frac{1}{4} = \frac{1}{52}$
3. $1 - \frac{1}{13} = \frac{12}{13}$
4. $\frac{1}{13} + \frac{1}{4} - \frac{1}{13} \cdot \frac{1}{4} = \frac{4}{52} + \frac{13}{52} - \frac{1}{13} \cdot \frac{1}{4} = \frac{16}{52}$

Recap

- Intersections - “AND” - multiply
  $P(E \cap F) = P(E) \cdot P(F)$ when independent
- Unions - “OR” - add
  $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
- Complements - “NOT” - subtract from 1
  $P(\text{not } E) = 1 - P(E)$