### Preliminaries and Objectives

**Preliminaries**
- Techniques of Counting
- Definition of Probability

**Objectives**
- Find probabilities of events combined using AND, OR, NOT

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#### Unions

Example: Roll two dice. What is the probability that the total of the two dice is either 7 or 11?

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- Event $E$ = total is 7, $P(E) = \frac{6}{36}$
- Event $F$ = total is 11, $P(F) = \frac{2}{36}$
- $P(E \text{ or } F) = P(E \cup F) = P(E) + P(F) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36}$
Conditional Probability

Sometimes the occurrence of an event changes our mind about the probability of another event.

\[ E = \text{roll a } \begin{array}{c} \text{a} \\ \text{b} \end{array} \]

\[ F = \text{roll } \geq 10 \text{ on two dice} \]

\[ P(E) = \frac{1}{6} \]

\[ P(F) = \frac{6}{36} \]

\[ P(F \mid E) = \frac{3}{6} \]

Independence

If \( P(F \mid E) = P(F) \), then \( E \) and \( F \) are independent.
If the occurrence of event $E$ has no effect on the occurrence of event $F$, then $E$ and $F$ are said to be \textbf{independent}.

When rolling two dice, what is the probability that both are $\boldsymbol{\bullet\bullet}$?

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When rolling two dice, what is the probability that both are $\boldsymbol{\bullet\bullet}$?

$E = \boldsymbol{\bullet\bullet}$

$F = \boldsymbol{\bullet\bullet}$

$P(E \text{ and } F) = P(E \cap F) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$P(E \text{ and } F) = P(E \cap F) = P(E) \cdot P(F)$

There is a 40% chance that it will rain today. What is the chance that it will not rain today?

\textbf{Answer}: $P(\text{no rain}) = 1 - P(\text{rain}) = 60\%$
Unions of Independent Events

When rolling two dice, what is the probability that at least one of the dice is \( \heartsuit \)?

\[
\begin{array}{cccccc}
\text{\( \diamondsuit \)} & | & \text{\( \clubsuit \)} & | & \text{\( \heartsuit \)} & | \\
2 & | & 3 & | & 4 & | \\
3 & | & 4 & | & 5 & | \\
4 & | & 5 & | & 6 & | \\
5 & | & 6 & | & 7 & | \\
6 & | & 7 & | & 8 & | \\
7 & | & 8 & | & 9 & |
\end{array}
\]

\[
P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}
\]

Examples

When picking a card from a standard deck, what is the probability that ...

1. the card is either a \( \spadesuit \) or \( \heartsuit \)?
2. the card is 7 and \( \heartsuit \)?
3. the card is not a King?
4. the card is either a 7 or a \( \heartsuit \)?

\[
\begin{array}{c}
\text{K} \spadesuit & | & \text{K} \heartsuit & | & \text{K} \clubsuit & | & \text{K} \diamondsuit \\
\text{Q} \spadesuit & | & \text{Q} \heartsuit & | & \text{Q} \clubsuit & | & \text{Q} \diamondsuit \\
\text{J} \spadesuit & | & \text{J} \heartsuit & | & \text{J} \clubsuit & | & \text{J} \diamondsuit \\
10 \spadesuit & | & 10 \heartsuit & | & 10 \clubsuit & | & 10 \diamondsuit \\
9 \spadesuit & | & 9 \heartsuit & | & 9 \clubsuit & | & 9 \diamondsuit \\
8 \spadesuit & | & 8 \heartsuit & | & 8 \clubsuit & | & 8 \diamondsuit \\
7 \spadesuit & | & 7 \heartsuit & | & 7 \clubsuit & | & 7 \diamondsuit \\
6 \spadesuit & | & 6 \heartsuit & | & 6 \clubsuit & | & 6 \diamondsuit \\
5 \spadesuit & | & 5 \heartsuit & | & 5 \clubsuit & | & 5 \diamondsuit \\
4 \spadesuit & | & 4 \heartsuit & | & 4 \clubsuit & | & 4 \diamondsuit \\
3 \spadesuit & | & 3 \heartsuit & | & 3 \clubsuit & | & 3 \diamondsuit \\
2 \spadesuit & | & 2 \heartsuit & | & 2 \clubsuit & | & 2 \diamondsuit \\
\text{A} \spadesuit & | & \text{A} \heartsuit & | & \text{A} \clubsuit & | & \text{A} \diamondsuit
\end{array}
\]

Examples

When picking a card from a standard deck, what is the probability that ...

1. the card is either a \( \spadesuit \) or \( \heartsuit \)?
2. the card is 7 and \( \heartsuit \)?
3. the card is not a King?
4. the card is either a 7 or a \( \heartsuit \)?

Answers:

1. \( \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \)
2. \( \frac{1}{13} \cdot \frac{1}{4} = \frac{1}{52} \)
3. \( 1 - \frac{1}{13} = \frac{12}{13} \)
4. \( \frac{1}{13} + \frac{1}{4} - \frac{1}{13} \cdot \frac{1}{4} = \frac{4}{52} + \frac{13}{52} - \frac{1}{13} \cdot \frac{1}{4} = \frac{16}{52} \)

Recap

- Intersections - “AND” - multiply
  \( P(E \cap F) = P(E) \cdot P(F) \) when independent
- Unions - “OR” - add
  \( P(E \cup F) = P(E) + P(F) - P(E \cap F) \)
- Complements - “NOT” - subtract from 1
  \( P(\text{not } E) = 1 - P(E) \)