Expected Value

UNIVERSITY OF MINNESOTA

Preliminaries and Objectives

Preliminaries

- Average
- Probability of events
- Sigma Notation
- Binomial Theorem

Objectives

- Calculate the Average Value (Expected Value) of a random variable

Calculating an Average

If during a 30-day month, you worked for 8 hours on 13 of those days, 6 hours on 3 of those days, 4 hours on 4 of those days and had 10 days off, how many hours per day did you work on average?

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Definition

If a random variable X, takes on possible values $v_1, v_2, v_3, \ldots$ which have probabilities $p_1, p_2, p_3, \ldots$ respectively, then the **expected value*** of X is

$$E(X) = v_1 \cdot p_1 + v_2 \cdot p_2 + v_3 \cdot p_3 + \ldots$$

$$E(X) = \sum_i v_i \cdot p_i$$

* Note: **Expected value** is also called **average value.** In statistics, it is referred to as the **mean.**

Example 1 - Two dice

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Example 1 - Two dice

k	$P(X = k)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

Example 1 - Two dice

$$
\begin{aligned}
E(X) &= (2)\frac{1}{36} + (3)\frac{2}{36} + (4)\frac{3}{36} + (5)\frac{4}{36} + (6)\frac{5}{36} + (7)\frac{6}{36} \\
&\quad + (8)\frac{5}{36} + (9)\frac{4}{36} + (10)\frac{3}{36} + (11)\frac{2}{36} + (12)\frac{1}{36} \\
&= 7
\end{aligned}
$$

Example 2 - Flipping 4 coins

k	$P(X=k)$
0	$\frac{1}{16}$
1	$\frac{4}{16}$
2	$\frac{6}{16}$
3	$\frac{4}{16}$
4	$\frac{1}{16}$

$$E(X) = (0)\frac{1}{36} + (1)\frac{4}{16} + (2)\frac{6}{16} + (3)\frac{4}{16} + (4)\frac{1}{16} = 2$$

Example 3 - Is this game fair?

k	$P(X = k)$
- $1	.70
+ $1	.20
+ $4	.10

Example 3 - Is this game fair?

k	$P(X = k)$
- \$1	.70
+ \$1	.20
+ \$4	.10

$$E(X) = (-1)(.70) + (1)(.20) + (4)(.10) = -.10$$

Credits

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