Expected Value



Preliminaries

- Average
- Probability of events
- Sigma Notation
- Binomial Theorem

Objectives

• Calculate the Average Value (Expected Value) of a random variable

8(13) + 6(3) + 4(4)

$$\frac{8(13)+6(3)+4(4)}{30}=4.60$$

$$\frac{0(10)+8(13)+6(3)+4(4)}{30}=4.60$$

$$\frac{0(10)+8(13)+6(3)+4(4)}{30}=4.60$$

$$0 \cdot \left(\frac{10}{30}\right) + 8 \cdot \left(\frac{13}{30}\right) + 6 \cdot \left(\frac{3}{30}\right) + 4 \cdot \left(\frac{4}{30}\right) = 4.60$$

$$\frac{0(10)+8(13)+6(3)+4(4)}{30}=4.60$$

$$0 \cdot \left(\frac{10}{30}\right) + 8 \cdot \left(\frac{13}{30}\right) + 6 \cdot \left(\frac{3}{30}\right) + 4 \cdot \left(\frac{4}{30}\right) = 4.60$$

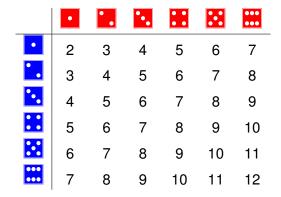
If a random variable X, takes on possible values $v_1, v_2, v_3, ...$ which have probabilities $p_1, p_2, p_3, ...$ respectively, then the **expected value**^{*} of X is

$$E(X) = v_1 \cdot p_1 + v_2 \cdot p_2 + v_3 \cdot p_3 + \dots$$

$$E(X) = \sum_i v_i \cdot p_i$$

* Note: **Expected value** is also called **average value**. In statistics, it is referred to as the **mean**.

Example 1 - Two dice



Example 1 - Two dice

k	P(X = k)
2	<u>1</u> 36
3 4	<u>2</u> 36
4	<u>3</u> 36
5 6 7	$\frac{4}{36}$
6	<u>5</u> 36
	<u>6</u> 36
8 9	<u>5</u> 36
9	$\frac{4}{36}$
10	<u>3</u> 36
11	<u>2</u> 36
12	<u>1</u> 36

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$$E(X) = (2)\frac{1}{36} + (3)\frac{2}{36} + (4)\frac{3}{36} + (5)\frac{4}{36} + (6)\frac{5}{36} + (7)\frac{6}{36} + (8)\frac{5}{36} + (9)\frac{4}{36} + (10)\frac{3}{36} + (11)\frac{2}{36} + (12)\frac{1}{36} = 7$$

Example 2 - Flipping 4 coins

k	P(X = k)
0	<u>1</u> 16
1	<u>4</u> 16
2	<u>6</u> 16
3	<u>4</u> 16
4	<u>1</u> 16

$$E(X) = (0)\frac{1}{36} + (1)\frac{4}{16} + (2)\frac{6}{16} + (3)\frac{4}{16} + (4)\frac{1}{16} = 2$$

Example 3 - Is this game fair?

k	P(X = k)
- \$1	.70
+ \$1	.20
+ \$4	.10

Example 3 - Is this game fair?

k	P(X = k)
- \$1	.70
+ \$1	.20
+ \$4	.10

E(X) = (-1)(.70) + (1)(.20) + (4)(.10) = -.10

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