

1. Expected Value

2. You should be familiar with the arithmetic concept of computing averages of a set of numbers and with the basics of probability. It may be helpful to know sigma notation and the binomial theorem

In this lesson, we will define **expected value** and perform calculations of expected values of random variables.

3. (a) The idea of computing an average is straightforward. You begin by adding up a sum of numbers, and dividing by the number of elements in the set. In this example, there are 30 days, and to compute the average number of hours worked, we merely need to add the total hours worked and divide by 30.
- (b) The total hours worked can be found by multiplying the number of hours worked by the number of days with that schedule. 8 hours for 13 days, 6 hours for 3 days, 4 hours for 4 days.
- (c) We then divide by the 30 days in the month to get the average.
- (d) Let's add in the 10 days you didn't work into the calculation. This doesn't affect the average since it doesn't affect the total hours.
- (e) It does allow us to write the expression using the fraction of days worked at a certain level. The same calculation can be seen as a number of hours times the fraction of days worked at that level.
- (f) The fractions should sum to 1 to account for all of the days. They can be seen as probabilities. The sum adds terms which are a value times a probability.
4. This is the definition of expected value, also called average value. To find the expected value of a random variable, multiply each value by the probability that it occurs, and add them up.
5. (a) Here is an example, rolling two six-sided dice, one red and one blue. There are 36 equally likely outcomes. For each, we calculate the total.
- (b) We can list the possible values of the random variable, and the associated probabilities.
- (c) To compute the expected value, we take each value times its probability and add them up. The average value on the roll of two dice is 7.
6. Here is a second example, flipping four coins. We find the probability of getting 0, 1, 2, 3 or 4 heads on the four flips, then multiply value times probability and find the sum.
7. (a) Here is a final example. You play a game where there is a 70% chance you lose \$1, a 20% chance you win \$1 and a 10% chance you win \$4. Do you want to play this game?
- (b) The expected value is negative ten cents, so in the long run, you will lose money.