Binomial Probabilities - Part I

University of Minnesota

Preliminaries and Objectives

Preliminaries

- Basic Probability (AND, OR, NOT)
- Binomial Theorem expanding $(x + y)^n$
- Pascal's Triangle
- Combinations
- Random Variables

Objectives

- Calculate probabilities in successive trials with only two outcomes, either succeed or fail.

Flip one coin

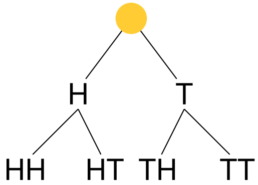
$$P(\text{heads}) = \frac{1}{2} \qquad P(\text{tails}) = \frac{1}{2}$$

Flip two coins

		Second Flip	
		Heads	Tails
First Flip	Heads	HH	HT
	Tails	TH	TT

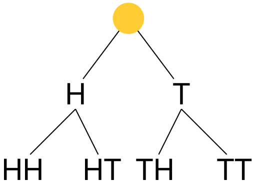
Flip two coins

		Second Flip	
		Heads	Tails
First Flip	Heads	HH	HT
	Tails	TH	TT



Flip two coins

		Second Flip	
		Heads	Tails
First Flip	Heads	HH	HT
	Tails	TH	TT



$$P(\text{HH}) = P(\text{HT}) = P(\text{TH}) = P(\text{TT}) = \frac{1}{4}$$

Flip two coins

		Second Flip	
		Heads	Tails
First Flip	Heads	HH	HT
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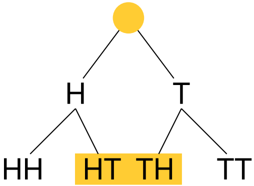

$$P(\mathrm{HH}) = P(\mathrm{HT}) = P(\mathrm{TH}) = P(\mathrm{TT}) = \frac{1}{4}$$

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		Heads	Tails
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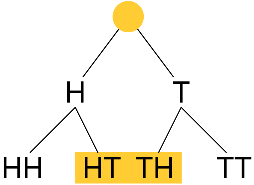


$$P(\text{two heads}) = \frac{1}{4}$$

$$P(\text{one head, one tail}) = \frac{2}{4}$$

$$P(\text{zero heads, two tails}) = \frac{1}{4}$$

Random Variable

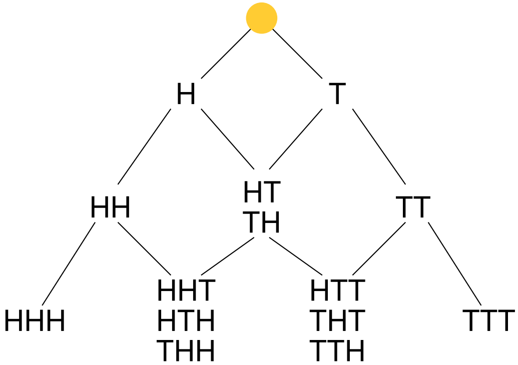
Let $H =$ the number of heads on two flips of a coin

$$P(H = 2) = \frac{1}{4}$$

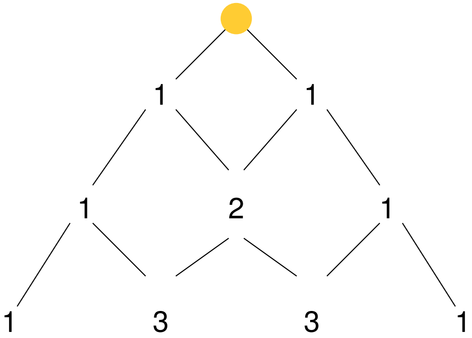
$$P(H = 1) = \frac{2}{4}$$

$$P(H = 0) = \frac{1}{4}$$

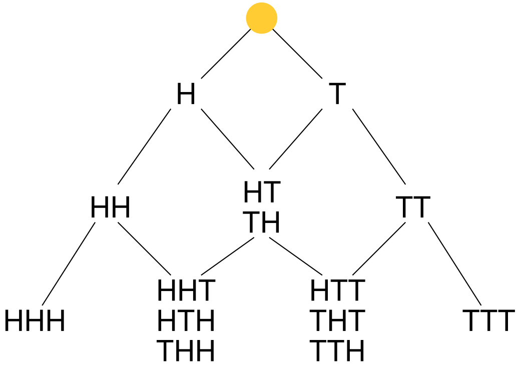
Flip three coins



Flip three coins



Flip three coins



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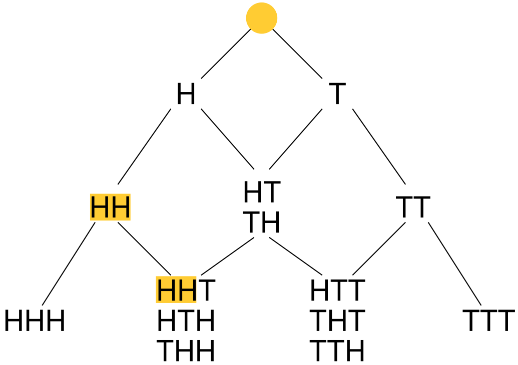

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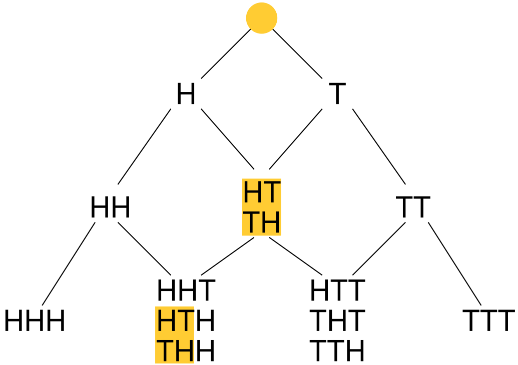
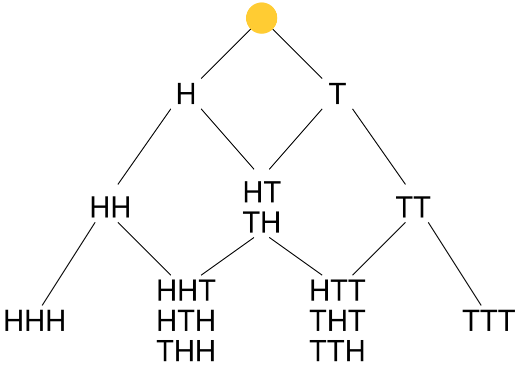


Flip three coins



$$P(H=3)=\frac{1}{8},\ \ P(H=2)=\frac{3}{8},\ \ P(H=1)=\frac{3}{8},\ \ P(H=0)=\frac{1}{8}$$

Pascal's Triangle - Flip five coins

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		5		10		10		5		1		
	1		6		15		20		15		6		1	
1		7		21		35		35		21		7		1

Pascal's Triangle - Flip five coins

$$
\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}
$$

$$P(H=5)=\frac{1}{32} \qquad P(H=4)=\frac{5}{32} \qquad P(H=3)=\frac{10}{32}$$

$$P(H=2)=\frac{10}{32} \qquad P(H=1)=\frac{5}{32} \qquad P(H=0)=\frac{1}{32}$$

General Formula for Coin Flips

Flip n coins, what is the probability that exactly k land heads?

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The numerator is the k^{th} number in row n of Pascal's Triangle.

1 5 10 10 5 1

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The denominator is 2^n

$$P(H = k) = \frac{{}_nC_k}{2^n} = \frac{C(n,k)}{2^n} = \frac{\binom{n}{k}}{2^n}$$

Example 1

Flip 7 coins, what is the probability that exactly 5 land heads?

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1 7 21 35 35 21 7 1

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1	7	21	35	35	21	7	1

$$P(H = 5) = \frac{21}{128} \approx 0.164$$

Credits

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