1. Binomial Probabilities - Part I

2. You should be familiar with the basic notions of probability, including combining events with AND, OR and NOT. It is also useful to know the Binomial Theorem, it's connection to Pascal's Triangle and counting combinations. Finally, the idea of random variable will be used. In this lesson, we will calculate probabilities in successive trials with two options, such as heads or tails when flipping coins.

3. We begin with the simplest case. When you flip one coin, there are two possibilities, each with probability one-half.

4. (a) When we flip two coins, we use the General Counting Principle to delineate the possibilities. We make two rows for the outcome of the first flip, and two columns for the outcomes of the second flip. Altogether, we have four possible outcomes.
   (b) Another representation of this process is the 'tree diagram'. Begin at the top, the first coin flip could be either heads or tails, and we follow one of those two branches. The second flip then can be either heads or tails.
   (c) Since there are four possibilities, the probability of each is 1/4.
   (d) Two of these possibilities, 'HT' and 'TH' have one head on the two flips.
   (e) If we are interested in counting only the number of heads, we can look at the event 'one head, one tail'. Two of the four outcomes fit that description, so the probability of getting one head and one tail is 1/2

5. We can also list this information using the idea of a random variable. Letting H be the number of heads, we can list the event 'one head, one tail' as the event $H = 1$.

6. (a) When we flip three coins, the tree diagram gains one extra level. At each step, each outcome splits in half, one string adding an H at the end, the other string adding a T at the end.
   (b) Notice that the number of outcomes containing a specified number of heads follows the pattern of Pascal's Triangle.
   (c) For example, there are three outcomes that have two heads on three flips.
   (d) We get this from the previous row. If we already have two heads, we add a tail.
   (e) If we had one head, then the next flip should be heads.
   (f) We can list the probabilities for the event based on the number of heads. Notice that the denominator is 2^3 because we have two possible choices on each coin flip, H or T, so each blank in the General Counting Principle is filled in with a 2, and there are three blanks for the three coin flips. The numerators are the numbers from the third row of Pascal's Triangle.

7. (a) To find the probabilities for flipping five coins, we will need the fifth row of Pascal's Triangle.

(b) All rows begin with a 1, the fifth row is the row that has a 5 next. Begin with the event getting no tails and all heads, and decrease the number of heads by one. The numerators are entries in Pascal's Triangle, 1, 5, 10, 10, 5, 1. The denominators are $2^5 = 32$.

8. (a) In general, to find the probability that when flipping n coins, exactly k land heads,

   (b) the numerator will be found from the n^{th} row of Pascal's Triangle. Here again is the example when $n = 5$.

   (c) Because if the symmetry, we can start with either all heads, or all tails. If we start with all tails, the leftmost entry is no heads, and then counts up. If you prefer, we can count down. There is one way to get all five heads.

   (d) 5 ways to get heads four times

   (e) 10 ways to get three heads

   (f) 10 ways to get two heads

   (g) 5 ways to get one head

   (h) and one way to get no heads

   (i) The denominator in each case is 2^n. The numerator is the k^{th} entry in row n of Pascal's Triangle, which is the number of combinations of size k from a set of size n. It is also referred to as 'n choose k', which is the binomial coefficient.

9. (a) Here is an example. What is the probability that we will get 5 heads in 7 coin flips?

   (b) We will need the seventh row of Pascal's Triangle

   (c) From the left, the entries are 7 heads, 6 heads, 5 heads, so the numerator we need is 21.

   (d) The denominator is 2^7 which is 128. The probability is approximately 16.4%