Binomial Probabilities - Part II

UNIVERSITY OF MINNESOTA

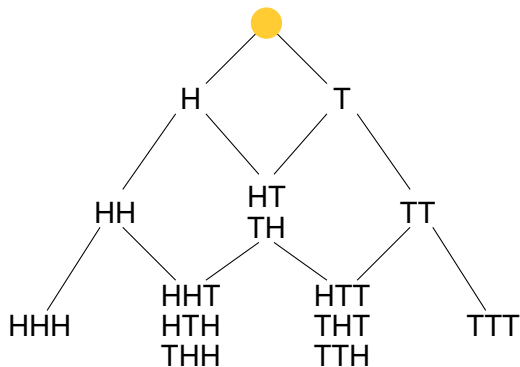## Preliminaries and Objectives

Preliminaries

- Basic Probability (AND, OR, NOT)
- Pascal's Triangle
- Combinations
- Binomial Probabilities $p = 1/2$
- Random Variables

Objectives

- Calculate probabilities in successive trials with only two outcomes, either succeed or fail.

Flip three coins

$$P(H = 3) = \frac{1}{8}, \;\; P(H = 2) = \frac{3}{8}, \;\; P(H = 1) = \frac{3}{8}, \;\; P(H = 0) = \frac{1}{8}$$
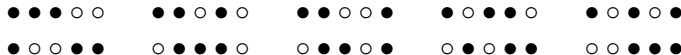
Unequally likely outcomes

A basketball player shoots 70% from the free throw line. Let $X =$ the number of shots made on two attempts.

$X =$	outcomes	probability
0	○○	$(.3)(.3) = .09$
1	●○ , ○●	$(.7)(.3) + (.3)(.7) = .42$
2	●●	$(.7)(.7) = .49$

A basketball player shoots 70% from the free throw line. Let
$X =$ the number of shots made on five attempts. What is the
probability that three of the five shots are made?

● ● ● ○ ○   ● ● ○ ● ○   ● ● ○ ○ ●   ● ○ ● ● ○   ● ○ ● ○ ●

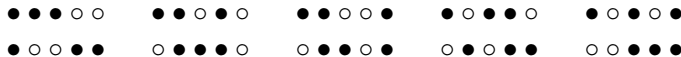● ○ ○ ● ●   ○ ● ● ● ○   ○ ● ● ○ ●   ○ ● ○ ● ●   ○ ○ ● ● ●
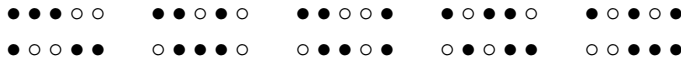
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● ○ ○ ● ●   ○ ● ● ● ○   ○ ● ● ○ ●   ○ ● ○ ● ●   ○ ○ ● ● ●

$P(● ● ● ○ ○) = P(● ● ○ ● ○) = \ldots$
$(.7)(.7)(.7)(.3)(.3) = (.7)(.7)(.3)(.7)(.3) = (.7)^3(.3)^2$

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● ○ ○ ● ●   ○ ● ● ● ○   ○ ● ● ○ ●   ○ ● ○ ● ●   ○ ○ ● ● ●

$P(\bullet \bullet \bullet \circ \circ) \qquad = P(\bullet \bullet \circ \bullet \circ) \qquad = \ldots$
$(.7)(.7)(.7)(.3)(.3) = (.7)(.7)(.3)(.7)(.3) = (.7)^3(.3)^2$

$P(X = 3) = \binom{5}{3}(.7)^3(.3)^2 = .3087$

Binomial Probability Formula

Let p = probability of success on a single trial.

Let $q = 1 - p$ = probability of failure on a single trial.

Let X be a random variable that counts the number of successes in n independent trials.

Binomial Probability

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

where $\binom{n}{k}$ is the kth entry in row n of Pascal's Triangle.

Find the probability that a 65% free throw shooter will make 3 of 4 attempts.

Exercise

Find the probability that a 65% free throw shooter will make 3 of 4 attempts.

Solution:

$$P(X = 3) = \binom{4}{3}(.65)^3(.35) = 4(.65)^3(.35) = .384475$$

Written by: Mike Weimerskirch

Narration: Mike Weimerskirch

Graphic Design: Mike Weimerskirch