1. Unions, Intersections, and Complements in Probability

2. You should be familiar with counting methods, such as the General Counting Principle, permutations, and combinations. It is also useful to know the Binomial Theorem. You should also be familiar with the basic definitions and terms of probability. In this lesson, we will calculate probabilities of events combined using the conjunctions AND, OR and NOT.

3. (a) The first idea is the union of two events, combined using the word 'OR'. It is easiest to calculate the probability if the two events are mutually exclusive, meaning that the two events can't happen at the same time. For example, when rolling two dice, you can't get both a total of 7 and a total of 11 at the same time. You can get one or the other, but not both. To get the total probability, we find the probabilities of the individual events separately, then add them together.

   (b) Here is a visual of the situation. There are six occurrences of a total of 7, shaded in pink, and two occurrences of a total of 11, shaded sky blue, for a total of 8 possibilities out of 36. We merely add the pink and blue boxes together.

   (c) Formally, the probability of E **or** F is the probability of E plus the probability of F.

4. (a) One thing that can complicate the situation is when one event influences the probability of another. For example, you are playing a game which you will win if you roll at least 10 on two dice. The probability of that event, event F, is 6/36.

   (b) There are three ways to roll 10,

   (c) two ways to roll 11

   (d) and one way to roll 12. When we learn of the outcome of one of the two dice, the blue die, we change our estimate of the likelihood. Now that we know that the blue die landed 6,

   (e) the probability of event F has increased. We call this **conditional probability**. What is the probability of F conditioned on the fact that event E happened? In this case, the probability of event F, getting a total of 10 now becomes a question of what happens on the second die. We already have 6, we just need 4 more, so three of the six possibilities are good. The probability of F, given that E happened, is 3/6, much better than the original 6/36.

   (f) Here is another look at the last example. The probability of getting a total of at least 10 when rolling two dice is 6 out of 36. Once we learn that the blue die landed 6, we only have 6 remaining possibilities, 3 of which give a total of at least 10.

5. Sometimes, E and F have nothing to do with one other. What did we roll on the red die? If I tell you that the blue die was 6, it gives us no information relevant to the red die. The probabilities don't change for the red die. If the probability of F, given E, is the same as it was without knowing E, then E and F are said to be independent. In terms of the two-dimensions grid, this means that all of the columns contain information that is consistent, not depending on the rows.

6. (a) When two events are independent, we can calculate the probability of two events both happening. For example, when rolling two dice, what is the probability that both are 5? The key word here is 'AND'. What is the probability that the blue die is 5 AND the red die is 5? We see here the one row that the outcome must be in, and the one column that the outcome must be in, giving us 1 success in 36 chances.

   (b) Formally, when combining probabilities of independent events using 'AND', we can find the probability of both by multiplying.

   (c) This can be seen by the General Counting Principle. To compute the denominator, we find the total possibilities by computing the area of the large rectangle. To find the numerator, we multiply the number of ways that E can occur by the number of ways that F can occur. This is the area of the darkest gray rectangle, once again found via the General Counting Principle by multiplying. This is the same procedure we use to multiply fractions.

7. The last word we need to understand is 'NOT'. We simply need to find the portion that is excluded. If there is a 40% chance of rain today, the remainder, 60% of the time, it won't rain.

8. We are now ready for a slightly more detailed formula for unions of independent events. When rolling two dice, what is the probability of getting a 5 on at least one of the two? This can be read using the word 'OR'. We need to get a 5 on the blue die OR a 5 on the red die. There are six outcomes in the blue row, there are six outcomes in the red column, but we have overcounted. When both dice are 5, we counted it in the blue row AND in the red column. We need to adjust for this by subtracting the double-counting, the intersection. The correct answer is to add the probabilities of E and F, then subtract the overlap. If E and F are mutually exclusive, then there is no overlap, and we have our original formula.

9. (a) Here are some examples. You may wish to pause the video to see if you can find these probabilities.

   (b) There are four suits, so 1/4 of the cards are spades and 1/4 of the cards are clubs. Since these are mutually exclusive, the word 'OR' means add, so we get 2/4.

   There are 13 possible values, of which '7' is one of them. There are 4 possible suits, of which hearts is one of them. The word 'AND' means multiply, so we get the probability 1/52. This makes sense because the 7 of hearts is one of the 52 cards in a deck.

   There are 13 ranks, of which king is one of the 13, so the other 12 ranks are not king

   In the last example, we need to account for the overlap. There are four 7s and 13 spades, which makes 17 possibilities, but we counted the 7 of spades twice, so we subtract the overlap to get 16 out of 52.

10. To recap: To combine probabilities using 'AND', we multiply the probabilities. The phrase 'both E and F' means E AND F.

To combine probabilities using ‘OR’ , we add the probabilities, subtracting the overlap if they are not mutually exclusive.

The word ‘NOT’ finds the remaining probability so that the two parts add to 100%