General Counting Principle

Preliminaries and Objectives

Preliminaries

- Multiplication
- Rectangles

Objectives

- Count the number of ways to make a selection with multiple parts.

Kids Menu

Main Dish	Side Dish
Hamburger	Yogurt
Hot Dog	Apple Slices
Pizza	Fries
Chicken Strips	String Cheese
Taco	

Kids Menu

	Yogurt	Apple Slices	Fries	String Cheese
Hamburger				
Hot Dog				
Pizza				
Chicken Strips				
Taco				

$$\text{Total choices} = 5 \cdot 4 = 20$$

Another Kids Menu

Main Dish	Side Dish	Drink
Hamburger	Yogurt	Milk
Hot Dog	Apple Slices	Juice
Pizza	Fries	
Chicken Strips	String Cheese	
Taco		

$$\text{Total choices} = 5 \cdot 4 \cdot 2 = 40$$

General Counting Principle

General Counting Principle

If there are m possible outcomes for a first event, and independently, n possible outcomes for a second event, then there are $m \cdot n$ possible pairs.

Example 1

A standard deck of cards has four suits, Clubs (♣), Diamonds (♦), Hearts (♥), and Spades (♠), and thirteen ranks (K,Q,J,10,9,8,7,6,5,4,3,2,A). Each card is identified by its rank and suit. (For example 2♣ or Q♥). How many cards are in a standard deck?

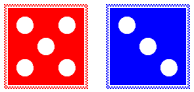
Answer: There are $4 \cdot 13 = 52$ cards in a standard deck

Standard Deck of Cards

K♠	K♥	K♦	K♣
Q♠	Q♥	Q♦	Q♣
J♠	J♥	J♦	J♣
10♠	10♥	10♦	10♣
9♠	9♥	9♦	9♣
8♠	8♥	8♦	8♣
7♠	7♥	7♦	7♣
6♠	6♥	6♦	6♣
5♠	5♥	5♦	5♣
4♠	4♥	4♦	4♣
3♠	3♥	3♦	3♣
2♠	2♥	2♦	2♣
A♠	A♥	A♦	A♣

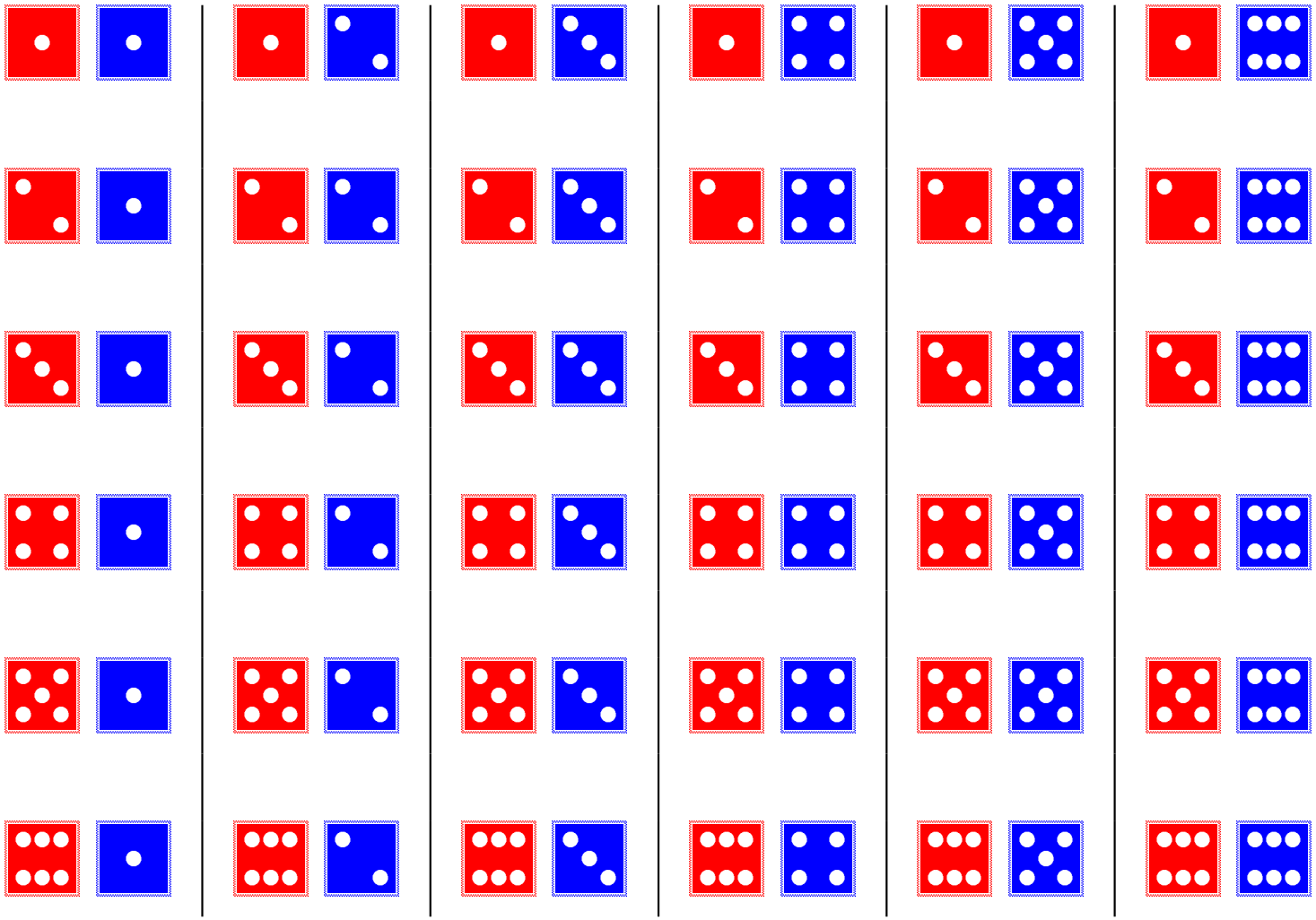
Example 2

How many different outcomes are possible when rolling two standard six-sided dice, one red and one blue?



Answer: $6 \cdot 6 = 36$

Two standard dice



Example 3 - 1947 Telephone Numbering Plan

Each telephone number in the United States and Canada was a ten-digit number with the following requirements:

Area Code: The first three digits were the area code. The first digit of the area code could not be a ‘0’ or a ‘1’. The second digit had to be a ‘0’ or a ‘1’. The third digit could be any digit form ‘0’ to ‘9’.

Exchange: The next three digits were the exchange. The first digit of the exchange could not be a ‘0’ or a ‘1’. The second digit could not be a ‘0’ or a ‘1’. The third digit could be any digit form ‘0’ to ‘9’.

The last four digits could be any digit form ‘0’ to ‘9’.

How many telephone numbers were possible?

Answer: $8 \cdot 2 \cdot 10 \cdot 8 \cdot 8 \cdot 10 \cdot 10^4 = 1,024,000,000$