Preliminaries

- General Counting Principle

Objectives

- Count the number of ways to make $n$ either/or choices
When you flip three coins, how many possible sequences of heads/tails can be formed?

\[ 2 \times 2 \times 2 = 8 \]

HHH  
HHT  
HTH  
HTT  

THH  
THT  
TTH  
TTT
How many TRUE/FALSE answer keys are possible on the three question quiz?

FFF  TFF
FFT  TFT
FTF  TTF
FTT  TTT
How many three digit binary numbers (numbers using only the digits ‘0’ and ‘1’) are possible?

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Other Examples

How many configurations of three switches that are either ON or OFF are possible? Answer: \(2^3 = 8\)

How many sequences of three black/white marbles are possible? Answer: \(2^3 = 8\)

How many hamburgers are possible, if in addition to the burger and the bun, you could also add ketchup, mustard and/or pickles?

- Plain
- Pickles

- Mustard
- Mustard, Pickles

- Ketchup
- Ketchup, Pickles

- Ketchup, Mustard
- Ketchup, Mustard, Pickles
Each keyboard key is encoded with an 8-bit binary string (an 8-digit number made of ‘0’s and ‘1’s). For example, the ASCII code for the exclamation point ‘!’ is 01000001. How many ASCII symbols are possible?

Answer: $2^8 = 256$
The general counting principle applies to repeated trials of other experiments. If there are $n$ possibilities for each trial, and $t$ trials, then the total number of outcomes is $t^n$.

Example: How many answer keys are possible for a multiple-choice exam with 6 questions, where each question has four choices (A, B, C, D)?

Answer:

$$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6 = 4096$$
1. If your hamburger can come with any of the following ingredients \{Ketchup, Mustard, Mayonnaise, Pickles, Lettuce, Tomatoes, Onions, Cheese\}, how many hamburgers are possible?

   Answer: \(2^8 = 256\)

2. How many ways are there to roll 5 six-sided dice?

   Answer: \(6^5 = 7776\)