

Combinations



Preliminaries and Objectives

Preliminaries

- General Counting Principle
- Permutations
- Factorial Notation

Objectives

- Count the number of ways to select k objects from a set of size n

Permutations

The number of ways to place k objects in order from a set of size n is

$$P(n, k) = \underbrace{(n)(n-1)(n-2)\dots(n-k+1)}_{k \text{ terms multiplied}} = \frac{n!}{(n-k)!}$$

Permutation Example



From a collection of five colored marbles, in how many ways, can you place three marbles in order?

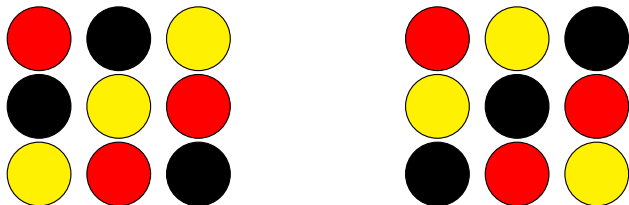
$$\underline{5} \times \underline{4} \times \underline{3} = 60 = \frac{5!}{(5-3)!}$$

What if we only care about the colors and not the order?

Combinations



From a collection of five colored marbles, in how many ways, can you place three marbles in order?



For each combination of three colors, there are $3!$ permutations of that one combination.

Combinations

In general, for each combination of k objects, there are $k!$ permutations of that one combination.

Therefore,

$$P(n, k) = k! \cdot C(n, k)$$

and

$$C(n, k) = \frac{P(n, k)}{k!} = \frac{n!}{k!(n-k)!}$$

where $C(n, k)$ denotes the number of combinations of k objects picked from a set of size n ,

Other Notations

$$C(n, k) = {}_n C_k = \binom{n}{k}$$

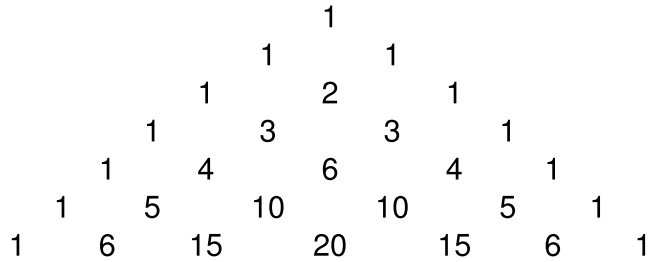
Connection to Pascal's Triangle

$\binom{n}{k}$ is the k^{th} entry in row n of Pascal's Triangle, where the n^{th} row begins

$$1 \quad n \quad \dots$$

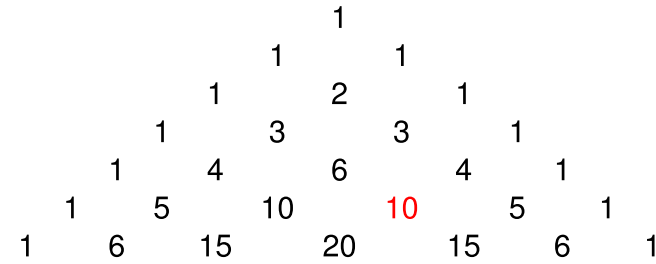
and the index k begins at zero, so the 0^{th} term of row n is 1, the first term is n etc.

Connection to Pascal's Triangle - Example



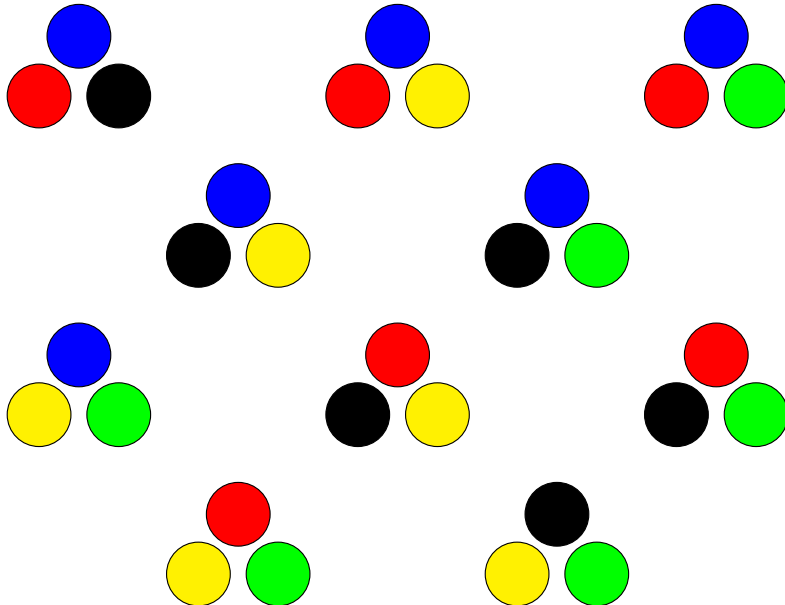
How many ways are there to select a combination of three marbles from a set of five?

Connection to Pascal's Triangle - Example



The number of ways to select a combination of three marbles from a set of five is 10.

Connection to Pascal's Triangle - Example



Combinations and Pascal's Triangle

The recursion for Pascal's Triangle is

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$$

This also works for combinations as in this example:

How many combinations of 3 marbles can be made from a set of 5 marbles?

How many combinations of 3 marbles can be made from a set of 5 marbles that contain the blue marble? ${}_4C_2 = 6$

How many combinations of 3 marbles can be made from a set of 5 marbles that don't contain the blue marble? ${}_4C_3 = 4$