

## Binary Counting



## Preliminaries and Objectives

### Preliminaries

- General Counting Principle

### Objectives

- Count the number of ways to make  $n$  either/or choices

## Coin Flips

When you flip three coins, how many possible sequences of heads/tails can be formed?

$$\underline{2} \times \underline{2} \times \underline{2} = 8$$

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

## True/False Tests

How many TRUE/FALSE answer keys are possible on the three question quiz?

FFF	TFF
FFT	TFT
FTF	TTF
FTT	TTT

## Binary Numbers

How many three digit binary numbers (numbers using only the digits '0' and '1') are possible?

000	100
001	101
010	110
011	111

## Other Examples

How many configurations of three switches that are either ON or OFF are possible? Answer:  $2^3 = 8$

How many sequences of three black/white marbles are possible? Answer:  $2^3 = 8$

How many hamburgers are possible, if in addition to the burger and the bun, you could also add ketchup, mustard and/or pickles?

Plain	Ketchup
Pickles	Ketchup, Pickles
Mustard	Ketchup, Mustard
Mustard, Pickles	Ketchup, Mustard, Pickles

## ASCII Codes

Each keyboard key is encoded with an 8-bit binary string (an 8-digit number made of '0's and '1's). For example, the ASCII code for the exclamation point '!' is 01000001. How many ASCII symbols are possible?

Answer:  $2^8 = 256$

## Extensions

The general counting principle applies to repeated trials of other experiments. If there are  $n$  possibilities for each trial, and  $t$  trials, then the total number of outcomes is  $t^n$ .

Example: How many answer keys are possible for a multiple-choice exam with 6 questions, where each question has four choices (A, B, C, D)?

Answer:

$$\underline{4} \times \underline{4} \times \underline{4} \times \underline{4} \times \underline{4} \times \underline{4} = 4^6 = 4096$$

## Exercises

- ① If your hamburger can come with any of the following ingredients {Ketchup, Mustard, Mayonnaise, Pickles, Lettuce, Tomatoes, Onions, Cheese }, how many hamburgers are possible?

Answer:  $2^8 = 256$

- ② How many ways are there to roll 5 six-sided dice?



Answer:  $6^5 = 7776$