## Permutations - Part I

University of Minnesota

## Preliminaries and Objectives

Preliminaries

- Multiplication
- General Counting Principle
- Recursion

Objectives

- Count the number of ways to put $n$ objects in order.
- Factorial Notation


## Two objects in order

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## FINISH

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## Two objects in order

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## Two objects in order

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## Two objects in order

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## Two objects in order

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## Two objects in order



## Two objects in order



## Two objects in order



## Three objects in order



## Three objects in order



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## Three objects in order



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## Three objects in order

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## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.


## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.


## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.


## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.


## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.


## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.


## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.


## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
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- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$
7 \cdot 6
$$

## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$
7 \cdot 6 \cdot 5
$$

## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$
7 \cdot 6 \cdot 5 \cdot 4
$$

## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$
7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot
$$

## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$
7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2
$$

## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$
7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1
$$

## Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$
7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1=5040
$$

## Factorial Notation

$$
7!=7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1=5040
$$

## Factorial Notation

$$
\begin{aligned}
& 7!=7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1=5040 \\
& n!=n \cdot(n-1) \cdot(n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1
\end{aligned}
$$

## Factorial Notation

$$
\begin{aligned}
& 7!=7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1=5040 \\
& n!=n \cdot(n-1) \cdot(n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \\
& n!=n \cdot(n-1)!
\end{aligned}
$$

## Example

1) brillig
A) $\ldots$
2) frumious
B) $\ldots$
3) manxome
C) $\ldots$
4) mimsy
D) $\ldots$
5) slithy E) ...

## Example

1) brillig
A) $\ldots$
2) frumious B) $\ldots$
3) manxome C) $\ldots$
4) mimsy D) $\ldots$
5) slithy E) ...

## Solution:

We need to put 5 letters in order, so the total number of ways to do this is $5!=5 \cdot 4 \cdot 3 \cdot 2 \cdot 1=120$

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