Permutations - Part I

Preliminaries and Objectives

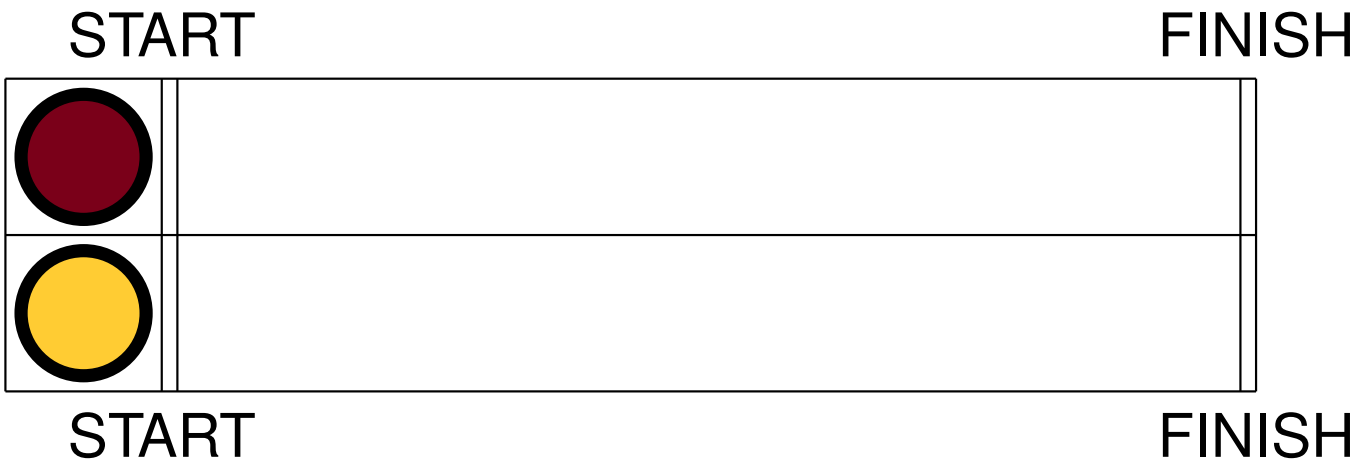
Preliminaries

- Multiplication
- General Counting Principle
- Recursion

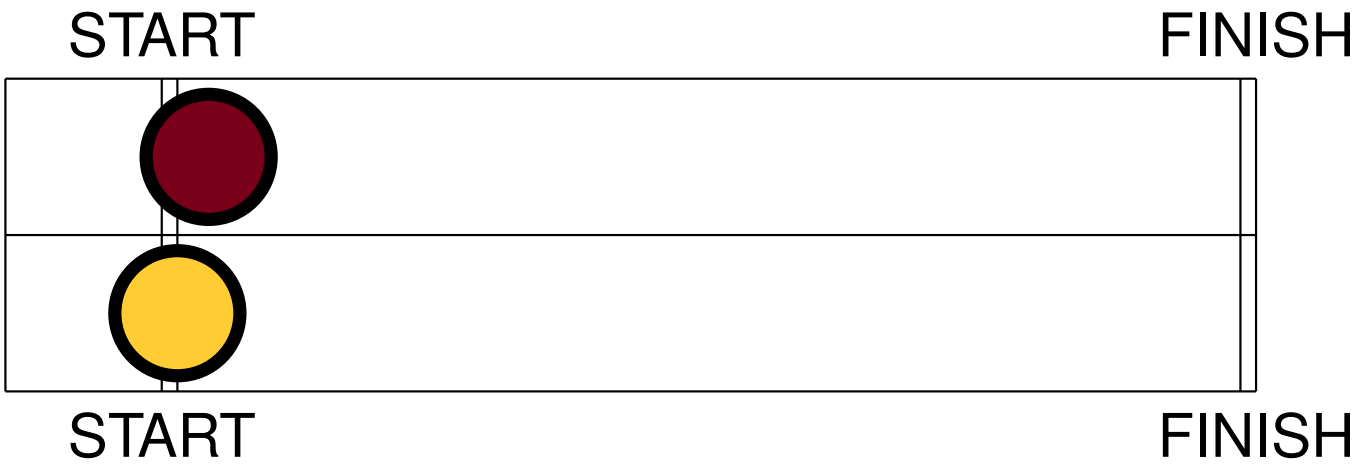
Objectives

- Count the number of ways to put n objects in order.
- Factorial Notation

Two objects in order

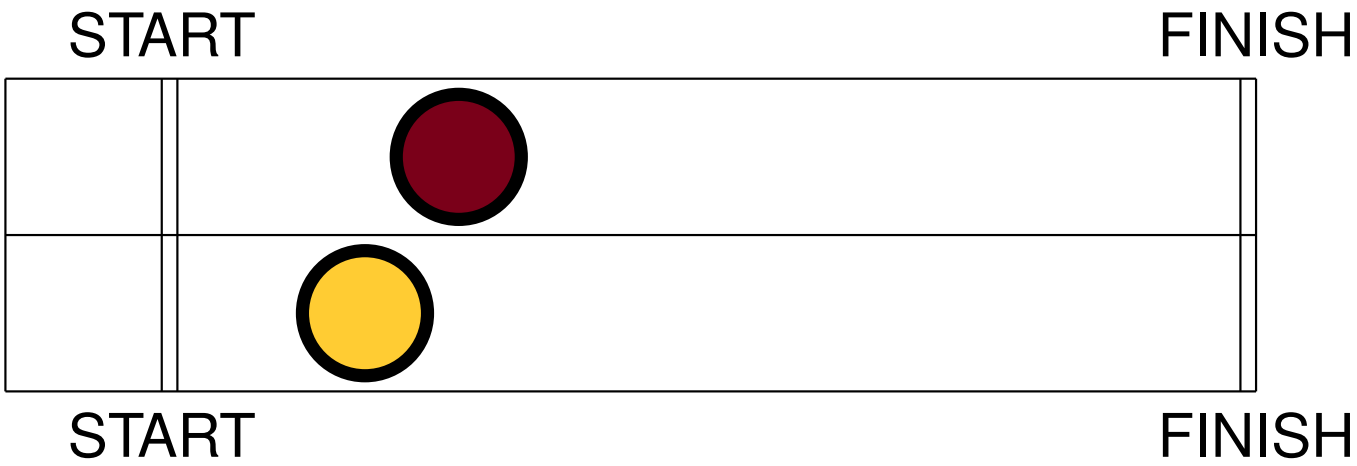


Two objects in order

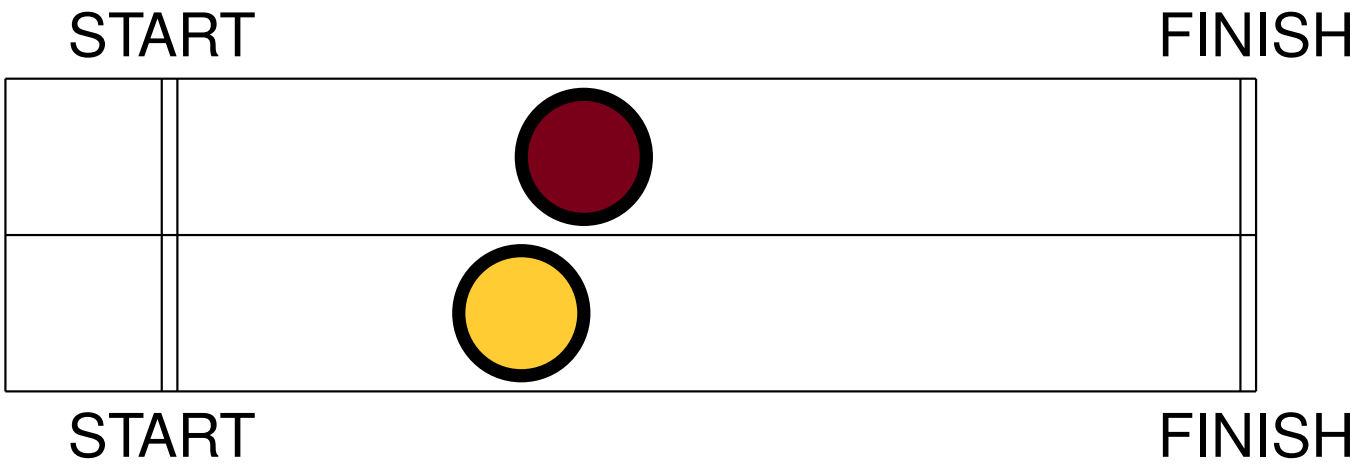


Two objects in order

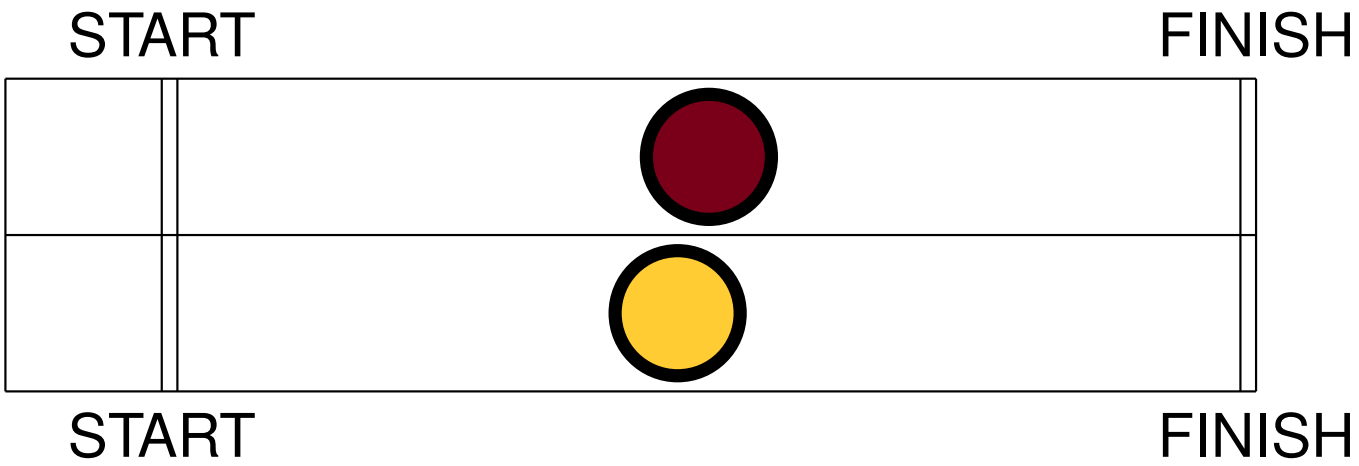
Two objects in order



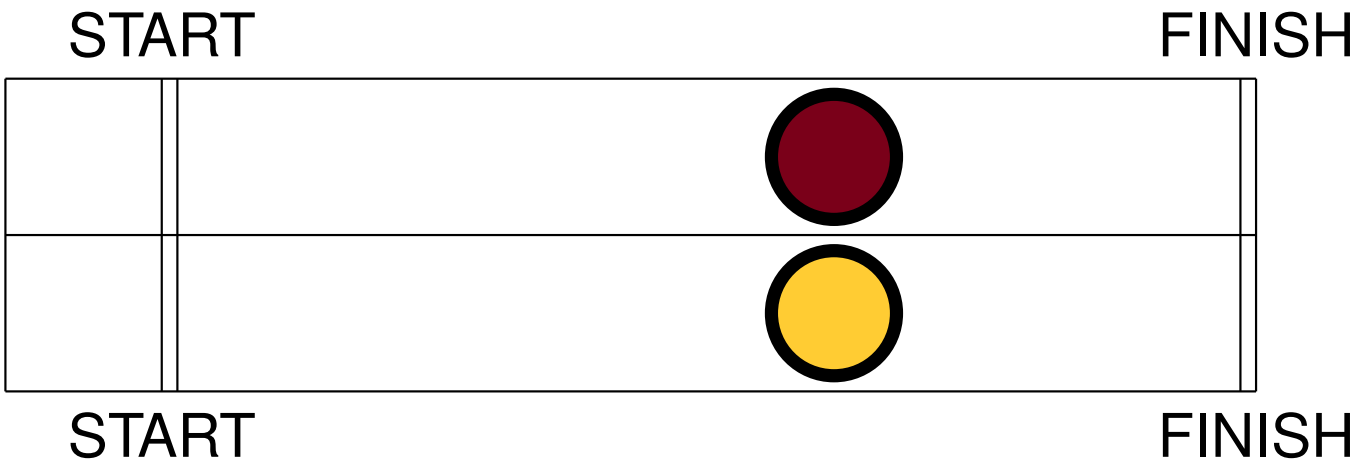
Two objects in order



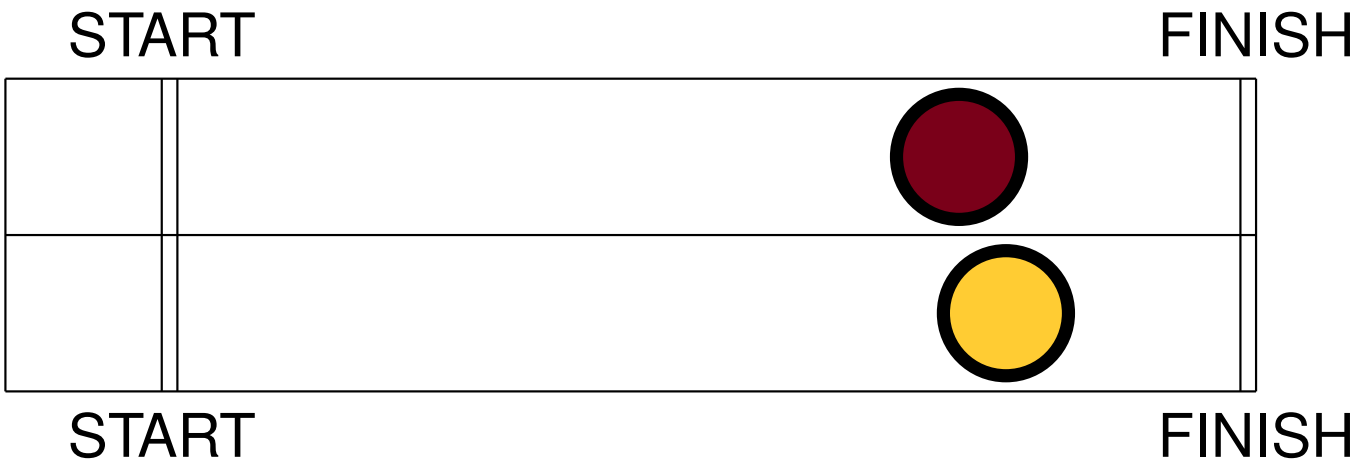
Two objects in order



Two objects in order



Two objects in order



Two objects in order

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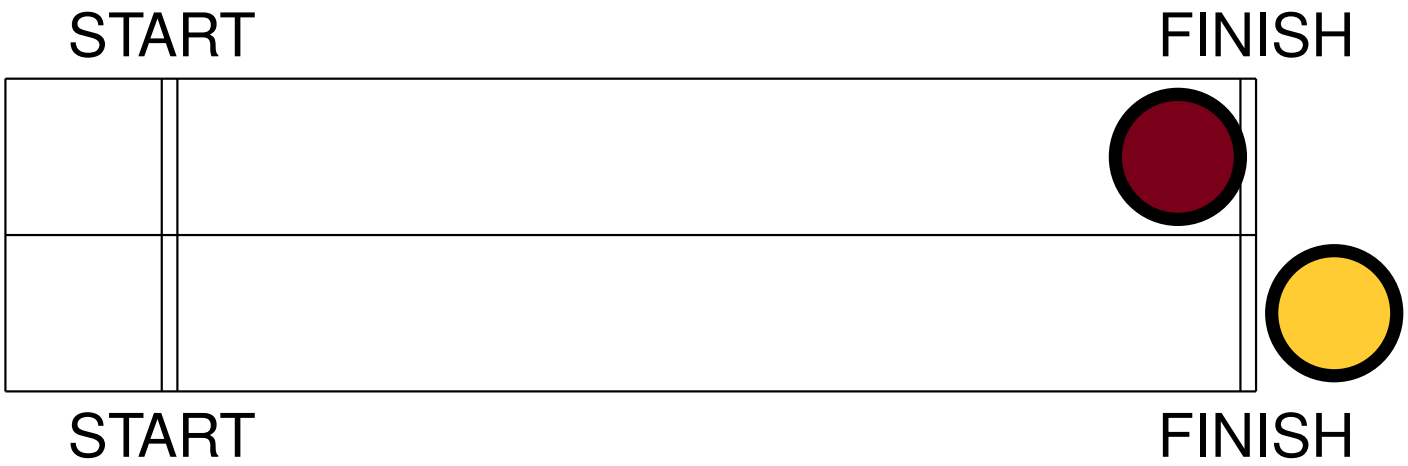
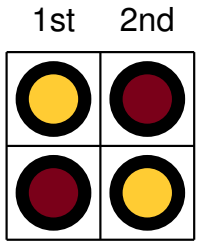
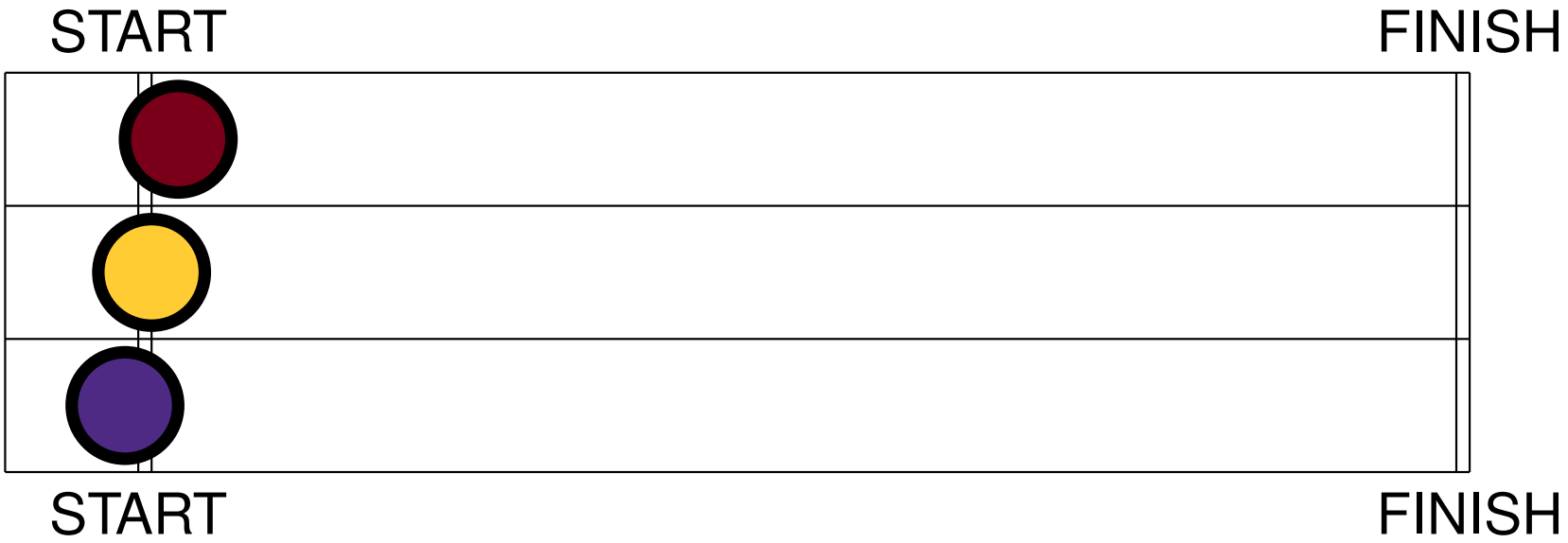




Three objects in order

Three objects in order



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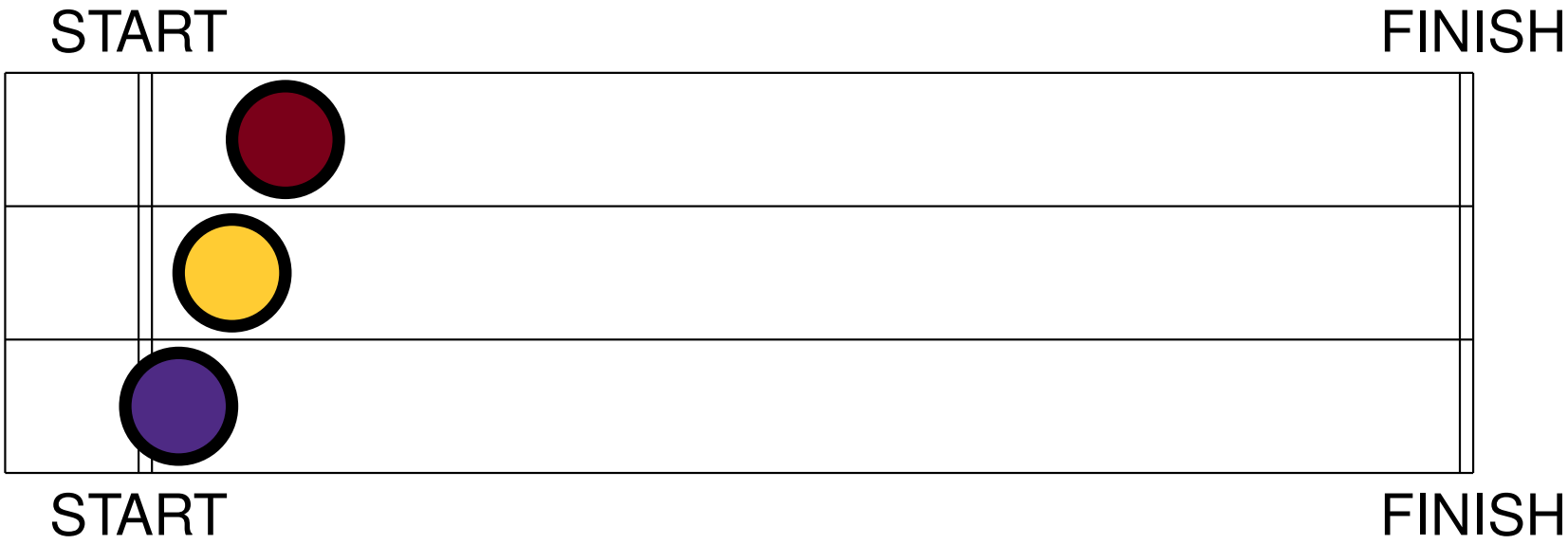

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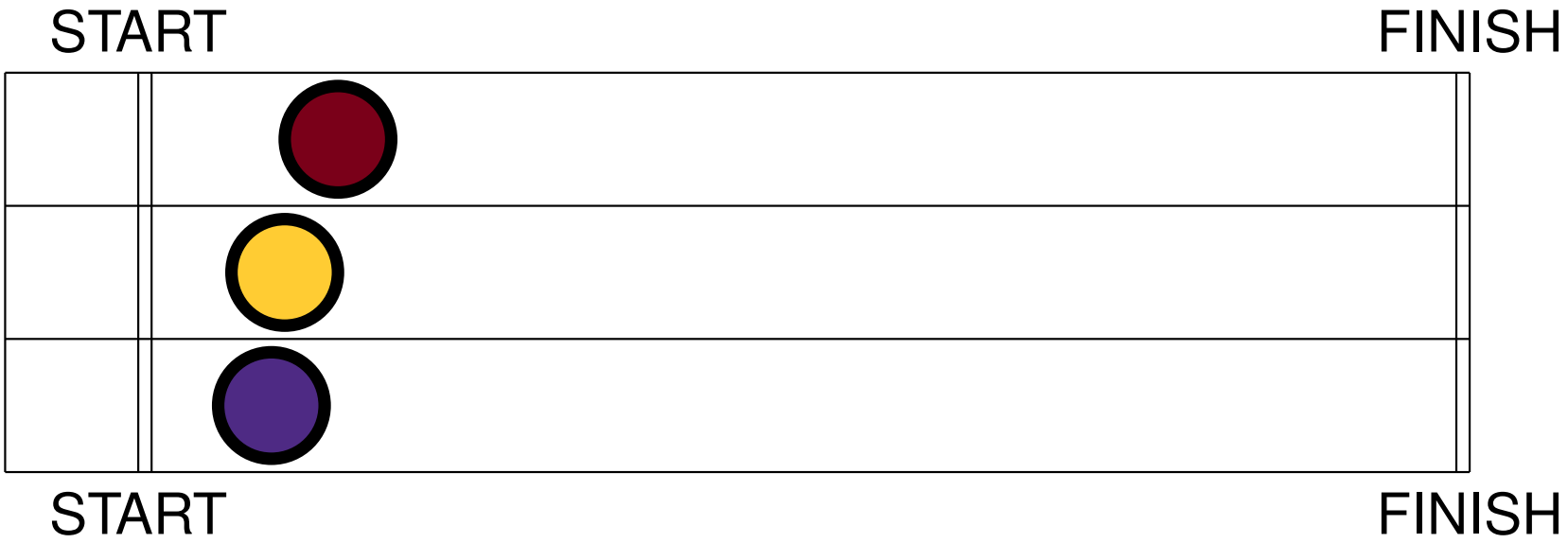
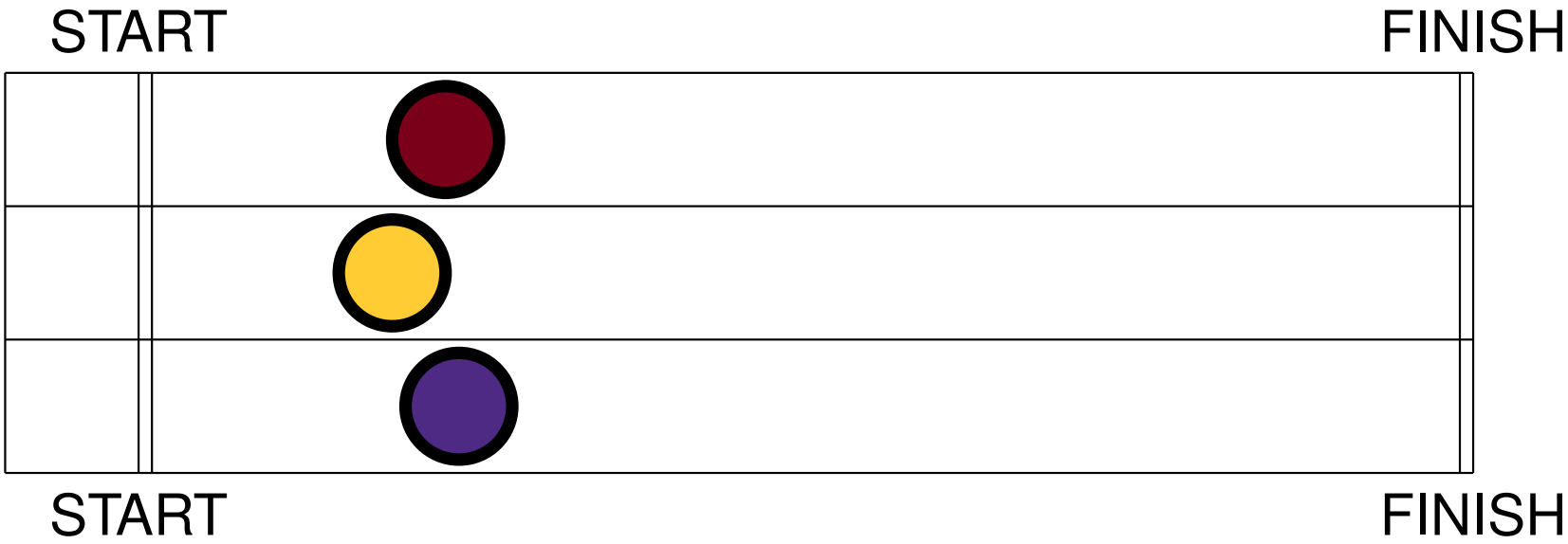


Three objects in order

Three objects in order



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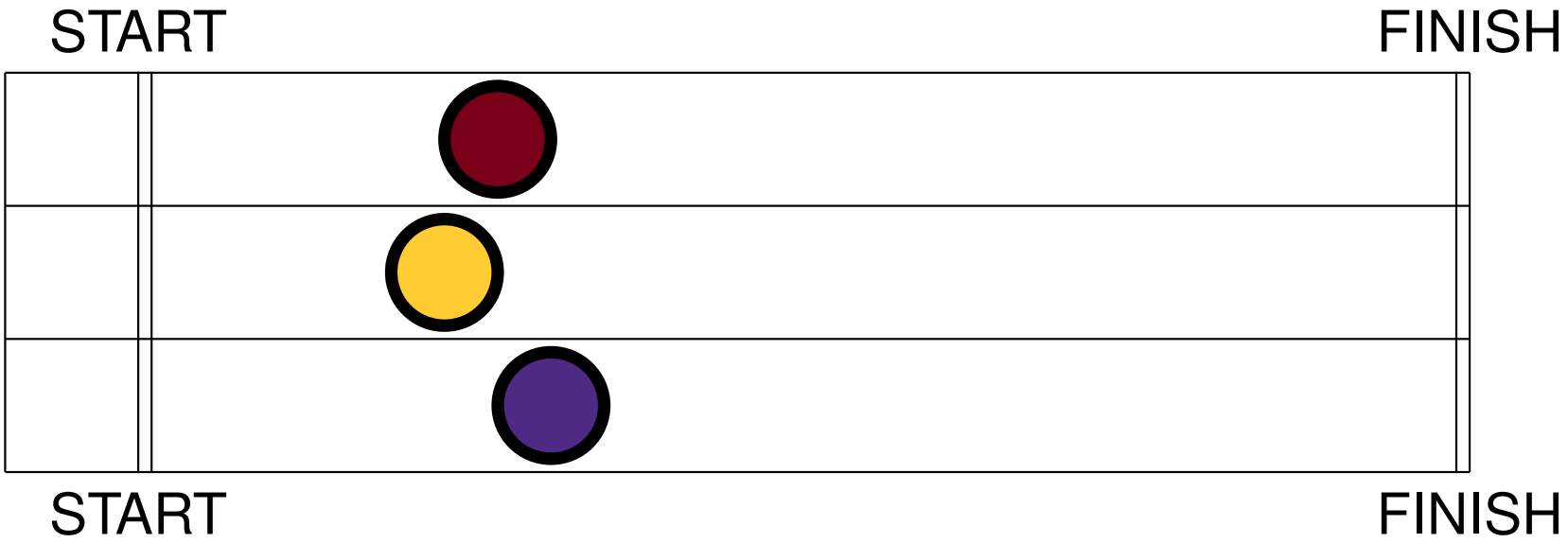
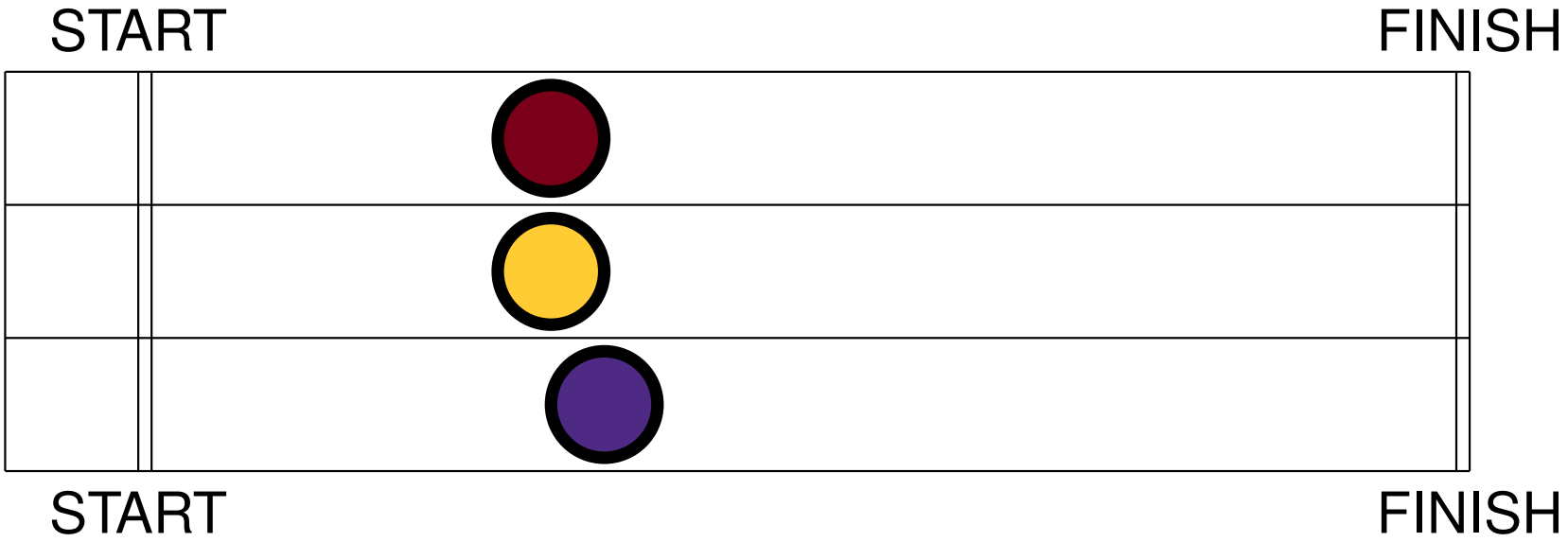


Three objects in order



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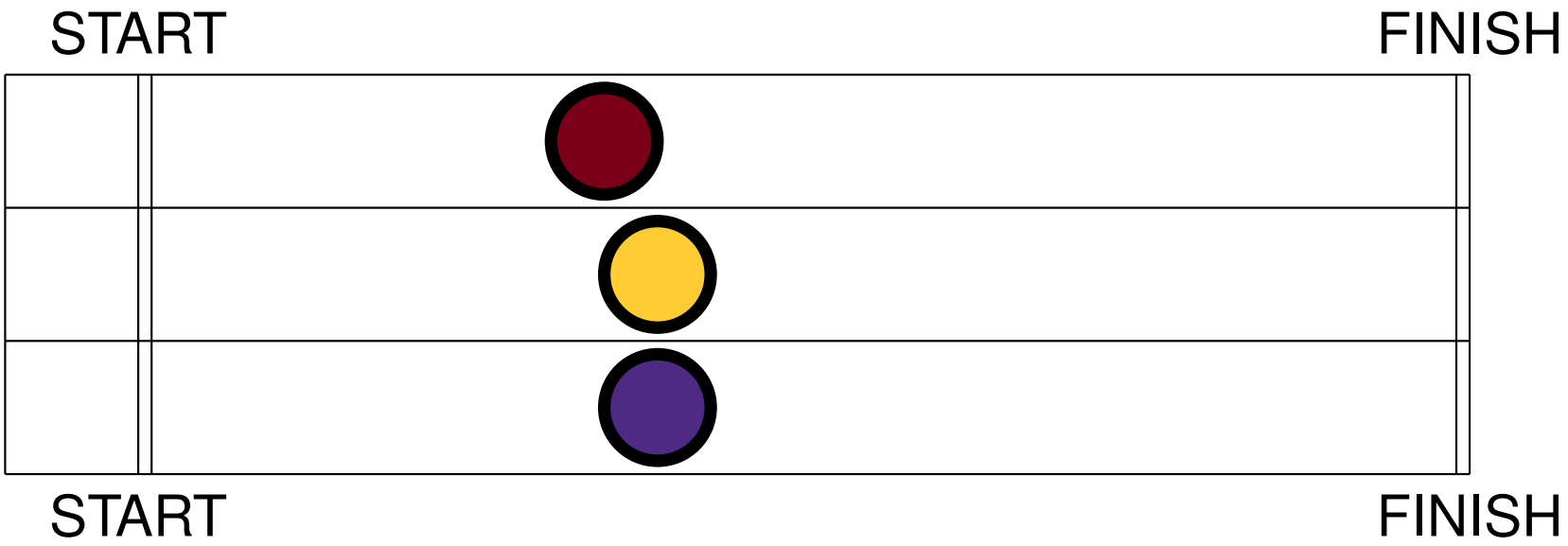

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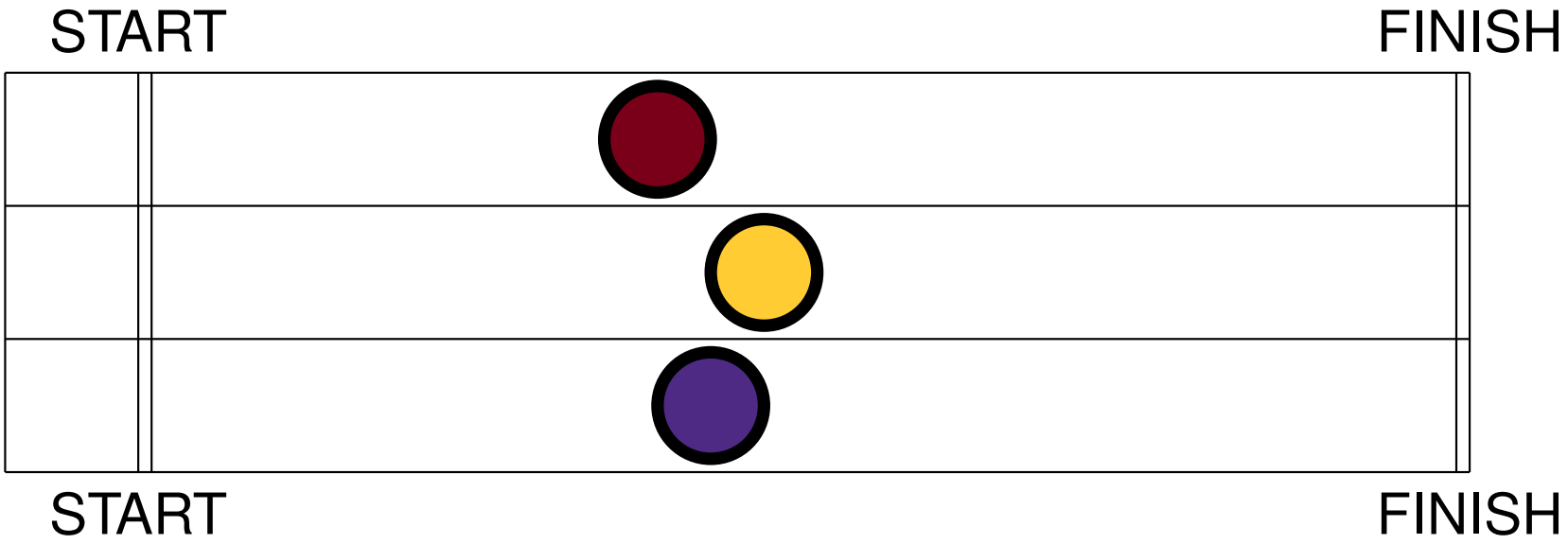

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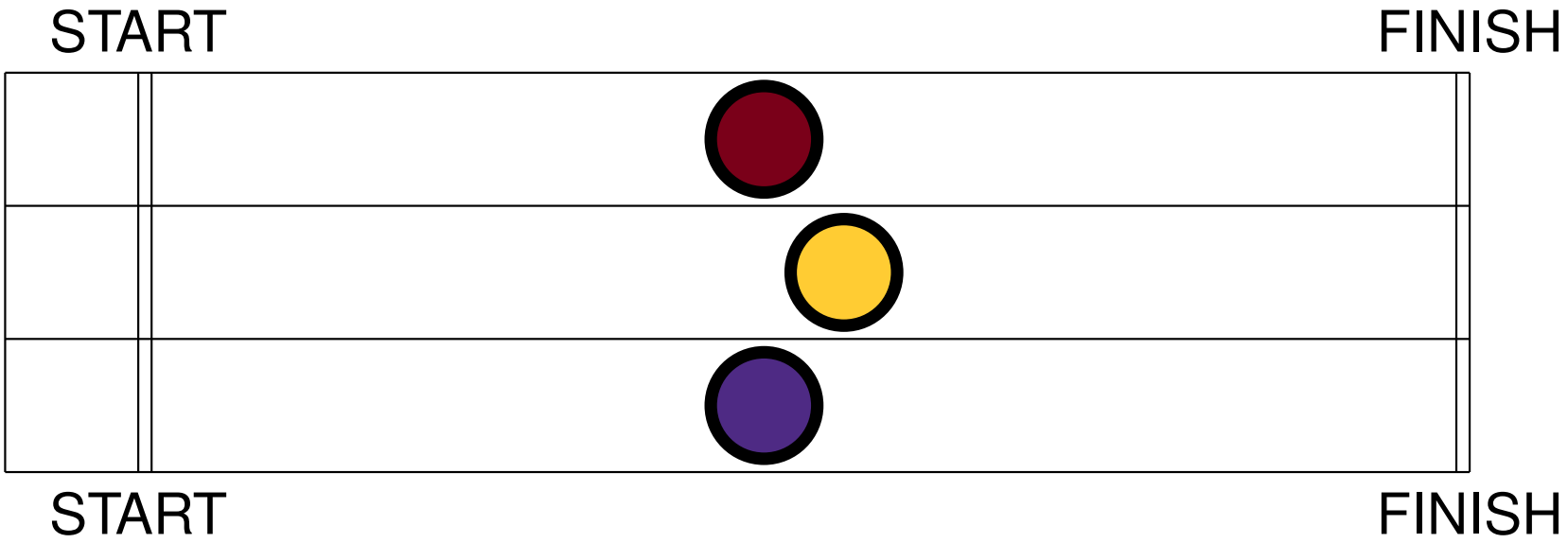

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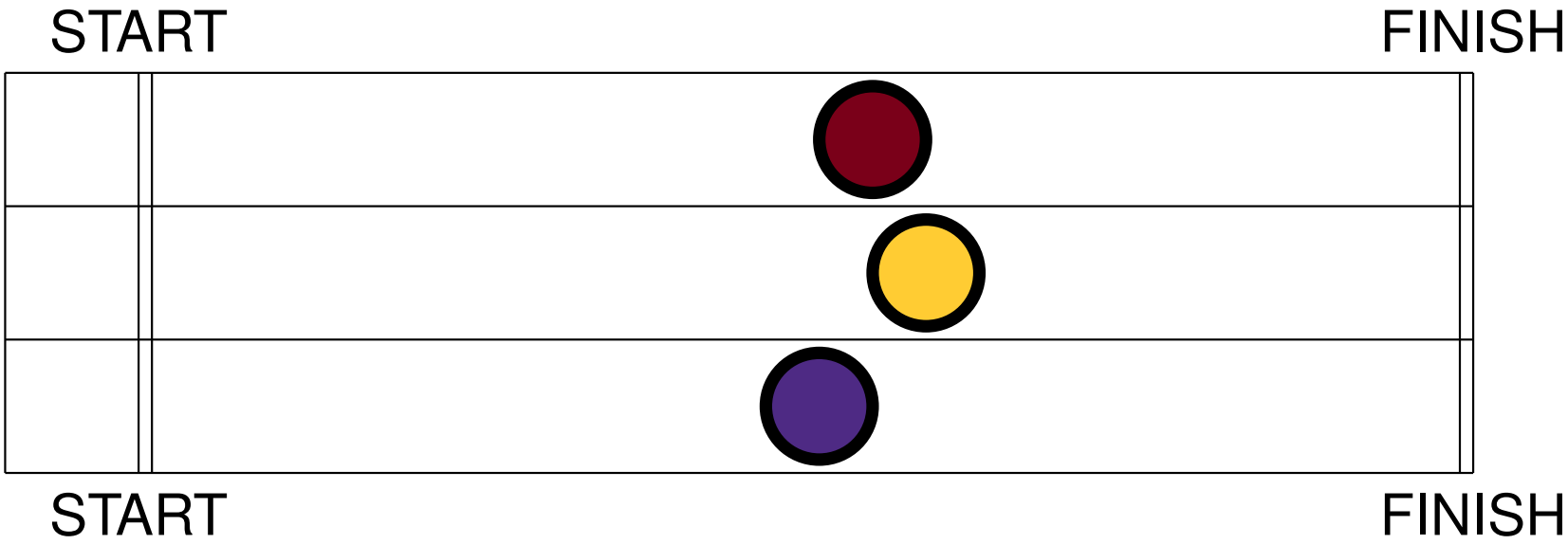

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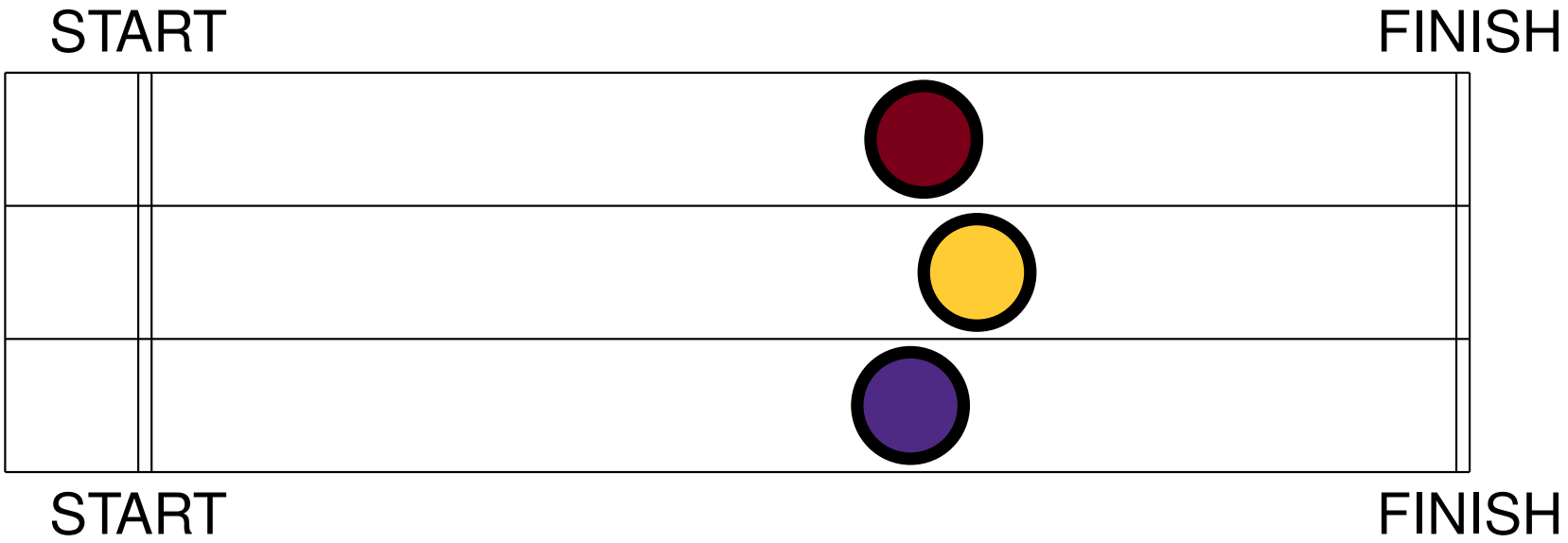

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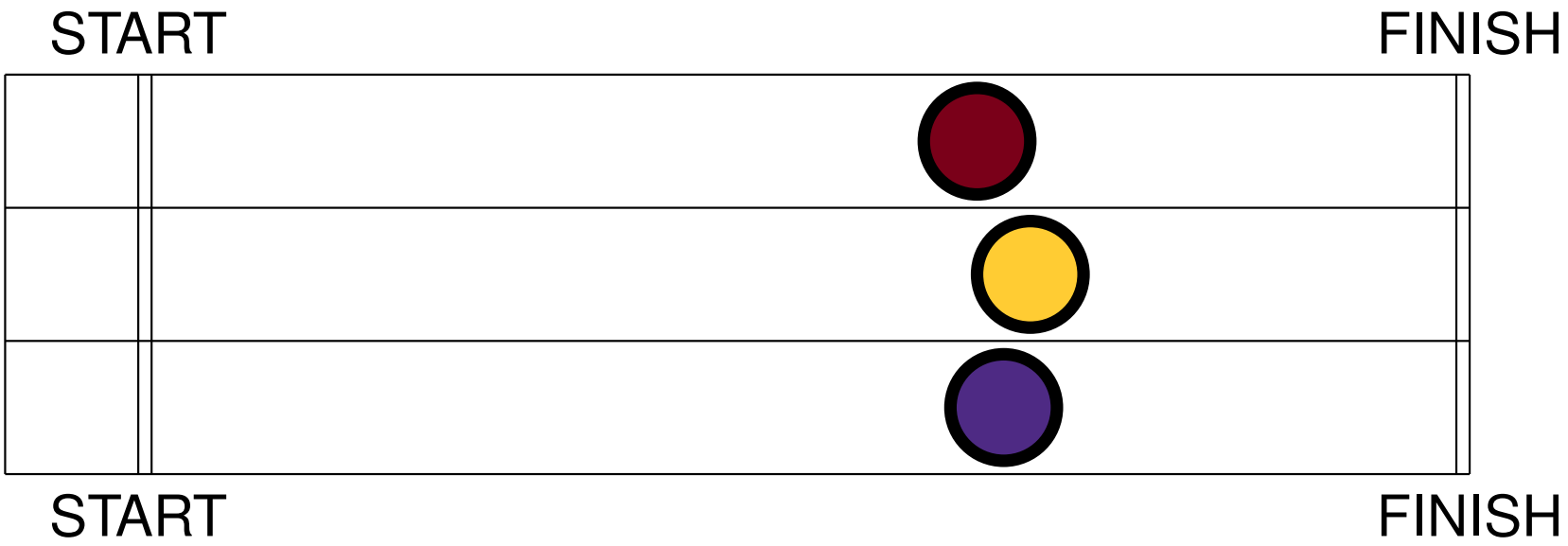

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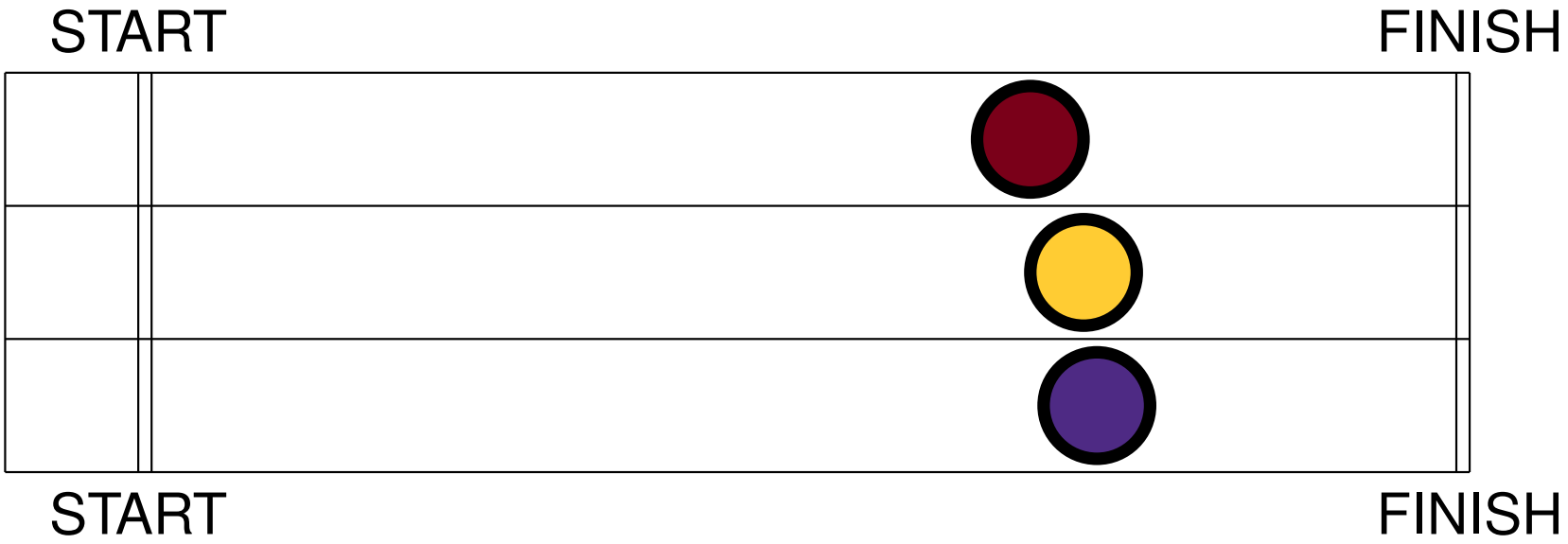

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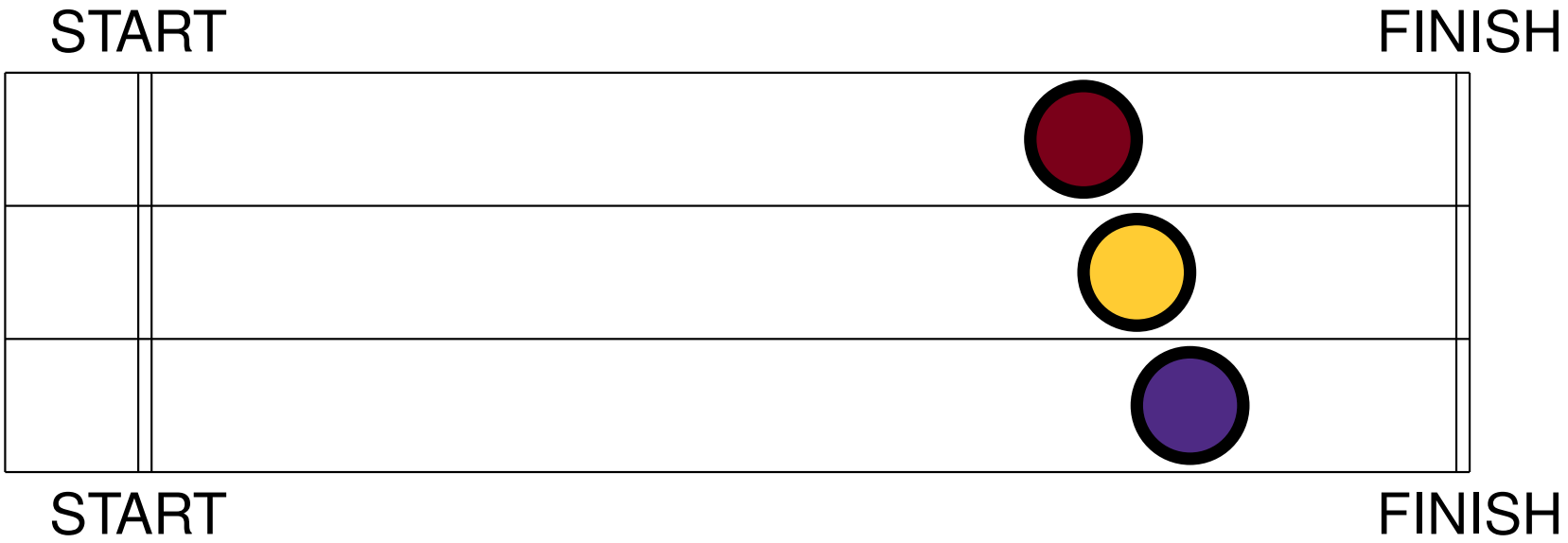

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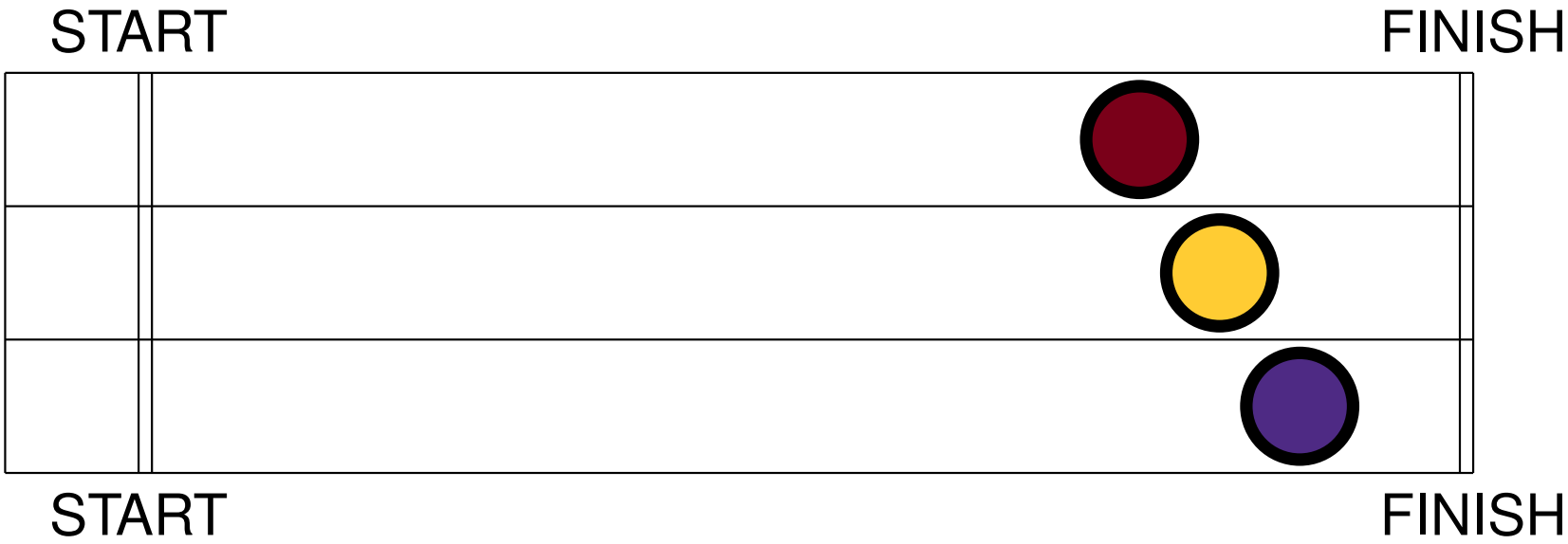

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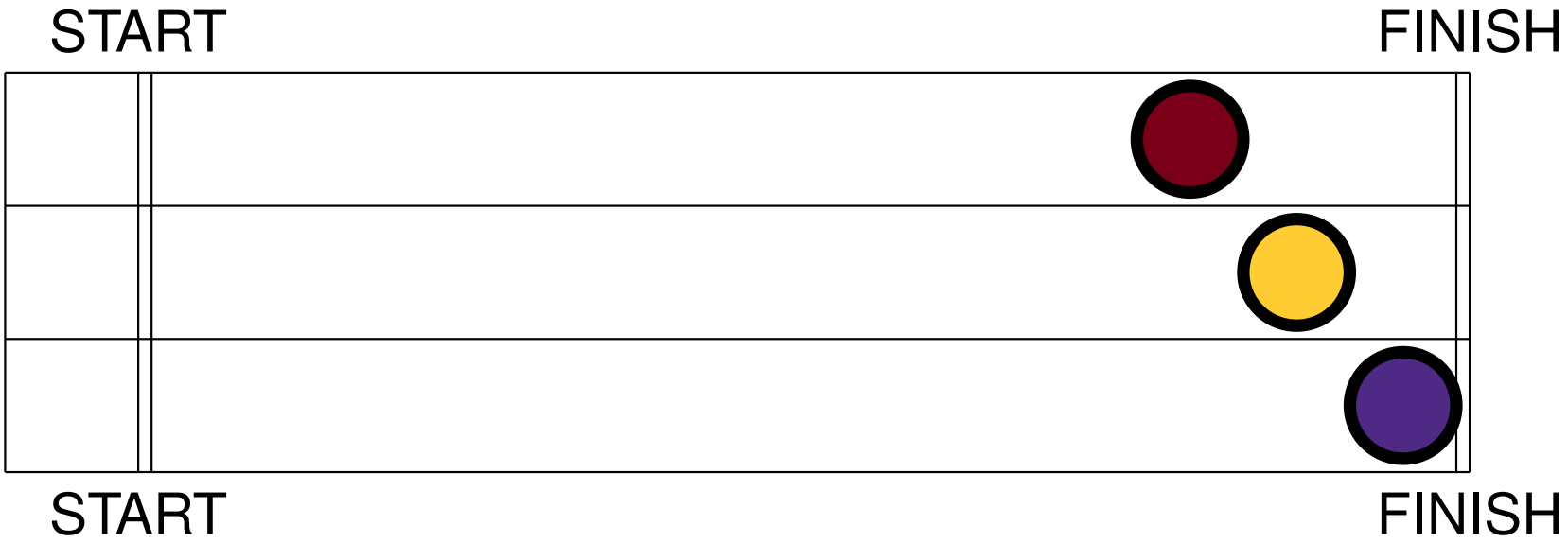

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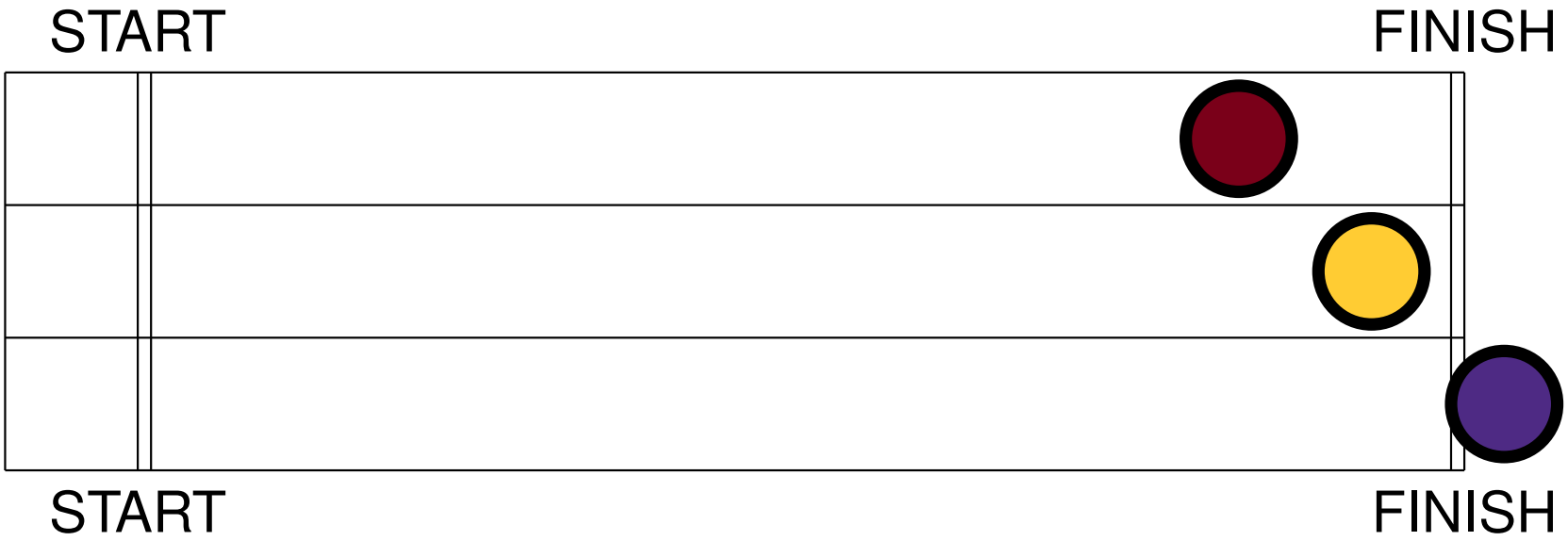

Three objects in order

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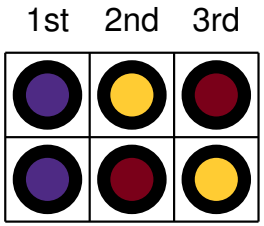

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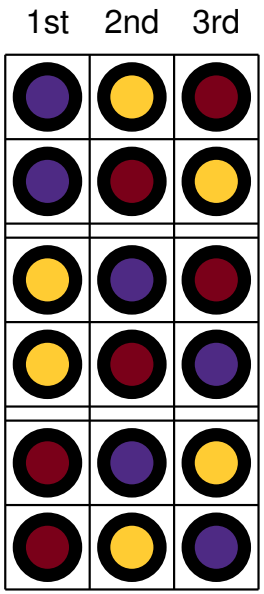

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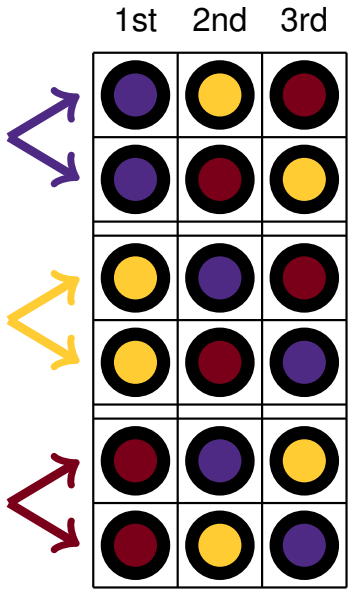

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$7 \cdot$$

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$7 \cdot 6 \cdot$$

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$7 \cdot 6 \cdot 5 \cdot$$

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot$$

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot$$

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot$$

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

Seven objects in order

- Pick the winner in one of 7 ways, then arrange the remaining runners.
- Pick the next runner in one of 6 ways, then arrange the remaining runners.
- Pick the next runner in one of 5 ways, then arrange the remaining runners.
- Pick the next runner in one of 4 ways, then arrange the remaining runners.
- Pick the next runner in one of 3 ways, then arrange the remaining runners.
- Pick the next runner in one of 2 ways, then arrange the remaining runners.
- Pick the last runner in the only way possible.

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

Factorial Notation

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

Factorial Notation

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

Factorial Notation

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

$$n! = n \cdot (n-1)!$$

Example

1)	brillig	A)	...
2)	frumious	B)	...
3)	manxome	C)	...
4)	mimsy	D)	...
5)	slithy	E)	...

Example

1)	brillig	A)	...
2)	frumious	B)	...
3)	manxome	C)	...
4)	mimsy	D)	...
5)	slithy	E)	...

Solution:

We need to put 5 letters in order, so the total number of ways to do this is $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

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