1. Permutations - Part I

2. There is very little that you need to know prior to this lesson, except for how to multiply. Understanding the General Counting Principle is helpful, but perhaps more formal than what is required in this lesson. Some familiarity with recursion is also helpful. In this lesson, you will learn to count the number of ways to put a collection of objects in order, and will learn factorial notation.

3. Suppose we have a race between two runners, one representing the maroon team and another representing the gold team. In how many different orders can they finish? In this case, the answer is simple, either maroon beats gold, or gold beats maroon, so there are two different orders.

4. (a) Now suppose there is a third runner from the purple team. In how many different orders can they finish? First, we must determine the winner. Suppose purple wins, we then must determine the places of the two remaining runners.
   (b) Notice that this is precisely the previous example, so we have two ways of arranging maroon and gold when purple has won.
   (c) There are three different runners that can win, and so our total list is three sets, one for each winner, and each set has two permutations based on the previous example. In total, there are six permutations, one represented in each row of the chart.

5. (a) We can repeat this process for any number of runners. To place seven runners in order, we first pick the winner in one of seven ways and repeat the process.
   (b) We next pick the second place runner in one of six ways. Each successive selection has one fewer choice until we have arranged all of the runners.
   (c) The total number of permutations of seven runners is seven times the number of ways to permute the remaining six, which was six times the number of ways to permute the remaining five, which was five times the number of ways to permute the remaining four... We continue this recursion until all runners have been placed. The total permutations of seven runners is 7 times 6 times 5 times 4 times 3 times 2 times 1.

6. (a) Since this process will be used often, it will be useful to have a shortcut notation. This notation is called **factorial** notation. Seven followed by an exclamation point is read *7 factorial*. It is 7 times 6 times 5 times 4 times 3 times 2 times 1.
   (b) For each positive integer n, $n!$ is n multiplied by each positive integer less than n.
   (c) Notice that after the n, the remainder is the product of all the integers from $n-1$ down to 1 which is $(n-1)!$. This fact is important for many of the simplifications performed in probability.

7. (a) Here is an example of permutations. Suppose you are answering a matching exam where you are given 5 words and asked to match them to their definitions. Unfortunately, you

didn't study and are forced to randomly guess. Because it is a matching exam, you do know that the answers to questions 1-5 are the letters A, B, C, D, and E in some order. In how many ways can you answer the exam?

(b) This is a permutation question. You are required to list five objects in order, so the answer is $5! = 120$