

1. Permutations - Part II
2. You should be familiar with the General Counting Principle, Permutations, and Factorial Notation. In this lesson, we will count the number of ways to put  $k$  out of  $n$  objects in order and notations for calculating permutations.
3. (a) Suppose we wish to place only three out of five marbles in order. In how many ways can this be done. Here are a few of the possibilities.
  - (b) To count the possibilities, we break this task into three choices according to the general counting principle.
  - (c) We have five ways to pick the marble that will be placed first ...
  - (d) four ways to pick the marble that will be placed second ...
  - (e) and three ways to pick the marble that will be placed third.
  - (f) To calculate the total possibility, we multiply according to the General Counting Principle.
4. In general, to place  $k$  out of  $n$  objects in order, we will need to multiply  $k$  numbers together, so we begin by making  $k$  blanks. In the first blank, we have  $n$  choices. In each successive blank, we have one fewer choice. Once all of the blanks are filled, multiply the numbers together according to the General Counting Principle.
5. (a) Here is the first example again.
  - (b) We will multiply three numbers together, beginning with 5, and going down by one each choice.
  - (c) Here is another approach based on factorials, we start with the numerator, as if we are placing all five marbles in order as a full permutation. We then realize that we don't need the last two, so we remove the last two factors by dividing by 2 times 1. Note that there were five marbles to start, and we were asked to arrange three of the five in order, so the factorial that we divide by is the number of marbles left over, in this case, 2.
6. (a) This can be generalized. Begin by placing all  $n$  in order, this is the  $n!$  in the numerator. We placed  $k$  in order, and need to remove the remaining  $(n - k)$  from the calculation.
  - (b) Several different notations are used for permutations. On many calculators, the permutation key has the letter 'P' followed by the two parameters,  $n$  and  $k$  in parentheses. Sometimes it is  $P(n, r)$  instead of  $P(n, k)$ . On other calculators and in print, the subscript  $n$  often comes before the 'P' and is read "n, P, k" or "n, P, r". In other sources, particularly in Europe, the  $k$  is a subscript on the  $n$ , which is by itself in parentheses. In all cases, it is calculated by the formula  $\frac{n!}{(n - k)!}$ . Again, recall that this is the same as writing  $k$  blanks, filling the blanks, starting with  $n$  and decreasing, then multiplying to get the final answer.