1. General Counting Principle

2. This is an introductory lesson in preparation of lessons in probability and has little background material. It relies on simple multiplication which ties conceptually to computing areas of rectangles. In this lesson, we will count the number of ways to make a selection with multiple parts.

3. In a cafeteria, kids can be served a meal by choosing one of five main dishes along with one of four sides. There will be hundreds of kids visiting today, and so we would like to prepare the meals in boxes ahead of their arrival so that we can be as efficient as possible serving them. How many different kinds of boxed meals will we need to prepare?

   We can think of this as a two step process, first, we prepare the main dish, then add the side. Each of the main dishes can be sent to one of four stations to add the appropriate side dish.

4. (a) It is easier to visualize the selections by listing the main dishes in rows and the sides in columns. Choosing a meal is now as simple as checking off one of the boxes in the grid.
   (b) Since there are five rows and four columns, there are $5 \times 4 = 20$ total choices of meals.

5. (a) Now let's add the drink. The calculation is similar. There are 20 meals that can go with milk, and 20 meals that can go with juice, so we have doubled our choices. This three part process can be thought of as a three-dimensional grid. The first page has the main dish rows and side dish columns with milk, the second page has the main dish rows and side dish columns with juice. On this two-dimensional screen, it is harder to represent this three-dimensional idea, but the principle is the same.
   (b) Our total choices are $5 \times 4 \times 2 = 40$. We could add a fourth choice and the same principle would apply, just multiply the choices at each stage.

6. Here is the General Counting Principle: If there are m possible outcomes for a first event and independently n possible outcomes for a second event, then there are $m \cdot n$ possible pairs.

7. (a) Here is an example. A deck of cards has four suits, each has 13 values, king, queen, jack on down to ace. How many cards are there in a deck?
   (b) To get the answer, multiply 4 times 13 to get 52 cards.

8. Here is a visual of the deck of cards. There are four columns for the four suits and 13 rows for the 13 values.

9. (a) Here is a second example; rolling two six-sided dice.
   (b) There are six choices for the red die and six for the blue die, so a total of 36.

10. There are six rows for the six different values for the red die and six columns for the six values of the blue die.

11. (a) Here is a final example. In 1947, the United States and Canada adopted a telephone numbering plan as follows. The first three digits were the area code. The area code could not start with '0' or '1'. '0' would dial the operator and '1' was an indicator that you were calling someone long distance. The second digit had to be a '0' or a '1' to distinguish them from exchanges.

    The exchanges could also not begin with '0' or '1', and their second digit also could not be '0' or '1'. Otherwise, all of the digits '0' - '9' were allowed. How many phone numbers were possible?

    (b) We need to multiply our ten steps together, 8 possible first digits, 2 possible second digits, 10 possible third digits, 8 possible fourth digits, 8 possible fifth digits, 10 possible sixth digits, and 10 possibilities for each of the last four digits, for a total of 1,024,000,000.