Binary Counting

University of Minnesota

Preliminaries and Objectives

Preliminaries

- General Counting Principle

Objectives

- Count the number of ways to make *n* either/or choices

When you flip three coins, how many possible sequences of heads/tails can be formed?

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$$\underline{\ 2\ } \times \underline{\ 2\ } \times \underline{\ 2\ } \qquad = 8$$

Coin Flips

When you flip three coins, how many possible sequences of heads/tails can be formed?

$$\underline{2} \times \underline{2} \times \underline{2} \quad = 8$$

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

True/False Tests

How many TRUE/FALSE answer keys are possible on the three question quiz?

FFF   TFF
FFT   TFT

FTF   TTF
FTT   TTT

Binary Numbers

How many three digit binary numbers (numbers using only the digits '0' and '1') are possible?

000	100
001	101
010	110
011	111

Other Examples

How many configurations of three switches that are either ON or OFF are possible? Answer: $2^3 = 8$

How many sequences of three black/white marbles are possible? Answer: $2^3 = 8$

How many hamburgers are possible, if in addition to the burger and the bun, you could also add ketchup, mustard and/or pickles?

Plain
Pickles

Ketchup
Ketchup, Pickles

Mustard
Mustard, Pickles

Ketchup, Mustard
Ketchup, Mustard, Pickles

ASCII Codes

Each keyboard key is encoded with an 8-bit binary string (an 8-digit number made of '0's and '1's). For example, the ASCII code for the exclamation point '!' is 01000001. How many ASCII symbols are possible?

Answer: $2^8 = 256$

Extensions

The general counting principle applies to repeated trials of other experiments. If there are *n* possibilities for each trial, and *t* trials, then the total number of outcomes is t^n.

Example: How many answer keys are possible for a multiple-choice exam with 6 questions, where each question has four choices (A, B, C, D)?

Answer:

$$\underline{\;4\;} \times \underline{\;4\;} \times \underline{\;4\;} \times \underline{\;4\;} \times \underline{\;4\;} \times \underline{\;4\;} = 4^6 = 4096$$

1 If your hamburger can come with any of the following ingredients {Ketchup, Mustard, Mayonnaisse, Pickles, Lettuce, Tomatoes, Onions, Cheese }, how many hamburgers are possible?

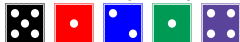**2** How many ways are there to roll 5 six-sided dice?

Exercises

1 If your hamburger can come with any of the following ingredients {Ketchup, Mustard, Mayonnaisse, Pickles, Lettuce, Tomatoes, Onions, Cheese }, how many hamburgers are possible?

Answer: $2^8 = 256$

2 How many ways are there to roll 5 six-sided dice?

Answer: $6^5 = 7776$